

section 2.7

2.7 1

$$434-a) |x| = 6$$

$$\begin{array}{lcl} +(x) = 6 & -(x) = 6 & \\ x = 6 & -x = 6 & \\ & -6 + x & -6 + x \\ \hline & -6 = x & \end{array}$$

$$434-b) |y| = -3$$

no solution

$$434-c) |z| = 0$$

$$\begin{array}{lcl} +(z) = 0 & -(z) = 0 & \\ z = 0 & z = 0 & \\ & z = 0 & \end{array}$$

$$436-a) |x| = 7$$

$$\begin{array}{lcl} +(x) = 7 & -(x) = 7 & \\ x = 7 & -x = 7 & \\ & -7 + x & -7 + x \\ \hline & -7 = x & \end{array}$$

$$436-b) |y| = -11$$

no solution

$$436-c) |z| = 0$$

$$z = 0$$

$$438) |2x-3| - 4 = 1$$

$$+4 +4$$

$$|2x-3| = 5$$

$$\begin{array}{lcl} +(2x-3) = 5 & -(2x-3) = 5 & \\ 2x-3 = 5 & -2x+3 = 5 & \\ +3 +3 & +2x -5 -5 +2x & \\ \hline 2x = 8 & -2 = 2x & \\ \frac{2x}{2} = \frac{8}{2} & \frac{-2}{2} = \frac{2x}{2} & \\ x = 4 & -1 = x & \end{array}$$

$$440) |3x-4| + 5 = 7$$

$$-5 -5$$

$$|3x-4| = 2$$

$$\begin{array}{lcl} +(3x-4) = 2 & -(3x-4) = 2 & \\ 3x-4 = 2 & -3x+4 = 2 & \\ +4 +4 & +3x -2 -2 +3x & \\ \hline 3x = 6 & 2 = 3x & \\ \frac{3x}{3} = \frac{6}{3} & \frac{2}{3} = \frac{3x}{3} & \\ x = 2 & \frac{2}{3} = x & \end{array}$$

$$442) 4|x-1| + 2 = 10$$

$$-2 -2$$

$$\frac{4|x-1|}{4} = \frac{8}{4}$$

$$|x-1| = 2$$

$$+(x-1) = 2$$

$$x-1 = 2$$

$$+1 +1$$

$$x = 3$$

$$-(x-1) = 2$$

$$-x+1 = 2$$

$$+x-2 -2 +x$$

$$-1 = x$$

$$444) \quad 3|4x-5|-4=11$$

$$\quad \quad \quad +4 \quad +4$$

$$\quad 3|4x-5|=15$$

$$\quad \quad \quad \frac{3}{3} \quad \frac{15}{3}$$

$$|4x-5|=5$$

$$+(4x-5)=5 \quad -(4x-5)=5$$

$$4x-5=5 \quad -4x+5=5$$

$$\quad +5 \quad +5 \quad \quad +4x-5-5+4x$$

$$\quad 4x=10 \quad \quad \quad 0=4x$$

$$\quad \frac{4}{4} \quad \frac{10}{4} \quad \quad \quad \frac{0}{4} \quad \frac{4x}{4}$$

$$x=\frac{5}{2} \quad \quad \quad 0=x$$

$$446) \quad -2|x-3|+8=-4$$

$$\quad \quad \quad -8 \quad -8$$

$$\quad -2|x-3|=-12$$

$$\quad \quad \quad \frac{-2}{-2} \quad \frac{-12}{-2}$$

$$|x-3|=6$$

$$+(x-3)=6 \quad -(x-3)=6$$

$$x-3=6 \quad -x+3=6$$

$$\quad +3 \quad +3 \quad \quad +x-6-6+x$$

$$x=9 \quad \quad \quad -3=x$$

$$448) \quad |\frac{3}{4}x-3|+7=2$$

$$\quad \quad \quad -7 \quad -7$$

$$\quad |\frac{3}{4}x-3|=-5$$

no solution

$$450) \quad |\frac{1}{2}x+5|+4=1$$

$$\quad \quad \quad -4 \quad -4$$

$$\quad |\frac{1}{2}x+5|=-3$$

no solution

$$452) \quad |3x-2|=|2x-3|$$

$$+(3x-2)=2x-3 \quad -(3x-2)=2x-3$$

$$3x-2=2x-3 \quad -3x+2=2x-3$$

$$\quad -2x+2 \quad -2x+2 \quad \quad +3x+3 \quad +3x+3$$

$$x=-1 \quad \quad \quad \frac{5}{5}=\frac{5x}{5}$$

$$\quad \quad \quad 1=x$$

$$454) \quad |6x-5|=|2x+3|$$

$$+(6x-5)=2x+3 \quad -(6x-5)=2x+3$$

$$6x-5=2x+3 \quad -6x+5=2x+3$$

$$\quad -2x+5 \quad -2x+5 \quad \quad +6x-3 \quad +6x-3$$

$$\quad 4x=8 \quad \quad \quad \frac{2}{8}=\frac{8x}{8}$$

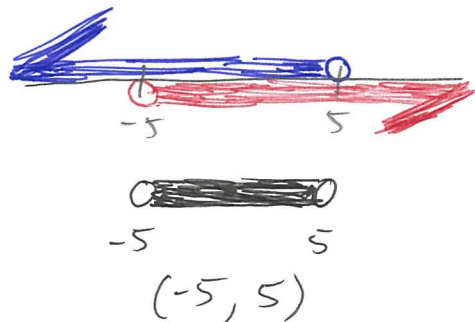
$$\quad \frac{4}{4} \quad \frac{8}{4} \quad \quad \quad \frac{1}{4}=x$$

$$x=2$$

for 456-467 solve using method for "and" 2.7 ³
 in section 2.6 { Absolute Value Inequalities with "less than" " $<$ " }

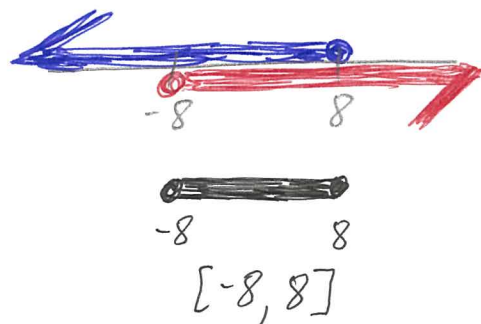
456) $|x| < 5$

$$\begin{array}{lcl} +(x) < 5 & \text{and} & -(x) < 5 \\ \downarrow & & \downarrow \\ x < 5 & \text{"both colors"} & -x < 5 \\ & & \underline{-5 + x \quad -5 + x} \\ & & -5 < x \end{array}$$



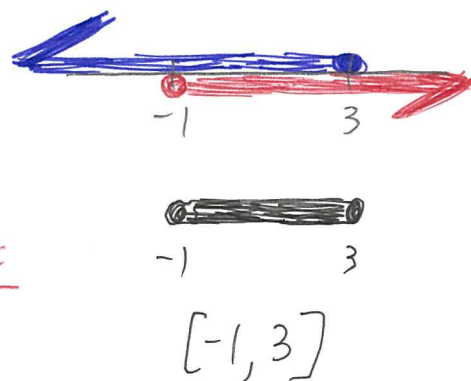
458) $|x| \leq 8$

$$\begin{array}{lcl} +(x) \leq 8 & \text{and} & -(x) \leq 8 \\ \downarrow & & \downarrow \\ x \leq 8 & \text{"both colors"} & -x \leq 8 \\ & & \underline{-8 + x \quad -8 + x} \\ & & -8 \leq x \end{array}$$



460) $|3x - 3| \leq 6$

$$\begin{array}{lcl} +(3x-3) \leq 6 & \text{and} & -(3x-3) \leq 6 \\ \downarrow & & \downarrow \\ 3x-3 \leq 6 & & -3x+3 \leq 6 \\ \underline{+3 \quad +3} & \text{"both colors"} & \underline{+3x-6 \quad -6+3x} \\ \frac{3x}{3} \leq \frac{9}{3} & & \underline{-3 \leq 3x} \\ x \leq 3 & & -1 \leq x \end{array}$$



462) $|2x+3| + 5 < 4$

$$\begin{array}{r} -5 \quad -5 \\ \hline |2x+3| < -1 \end{array}$$

contradiction
 no solution

Absolute valued items
 are positive, so it
 cannot be less than
 a negative number.

$$464) |4x - 3| < 1$$

2.7 4

$$+(4x - 3) < 1$$

and

$$-(4x - 3) < 1$$

$$4x - 3 < 1$$

↓

$$-4x + 3 < 1$$

$$\begin{array}{r} +3 \quad +3 \\ \hline 4x < 4 \end{array}$$

"both colors"

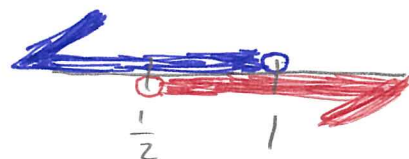
$$\begin{array}{r} +4x \quad -1 \quad -1 \quad +4x \\ \hline 2 < 4x \end{array}$$

$$\frac{4x}{4} < \frac{4}{4}$$

$$\frac{2}{4} < \frac{4x}{4}$$

$$x < 1$$

$$\frac{1}{2} < x$$



$$\frac{1}{2} \quad 1$$

$$(\frac{1}{2}, 1)$$

$$466) |x - 4| \leq -1$$

contradiction

no solution

for 468-479 solve using method for "or"
in section 2.6 {Absolute Value Inequality with "greater than" " $\geq$ "}

$$468) |x| > 3$$

$$+(x) > 3$$

or

$$-(x) > 3$$

$$x > 3$$

$$-x > 3$$

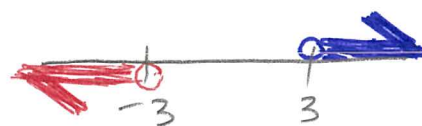
$$3 < x$$

any color

$$\begin{array}{r} -3 + x \quad -3 + x \\ \hline -3 > x \end{array}$$

$$-3 > x$$

$$x < -3$$



$$(-\infty, -3) \cup (3, \infty)$$

$$470) |x| \geq 2$$

$$+(x) \geq 2$$

or

$$-(x) \geq 2$$

$$x \geq 2$$

$$-x \geq 2$$

$$2 \leq x$$

any color

$$\begin{array}{r} -2 + x \quad -2 + x \\ \hline -2 \geq x \end{array}$$

$$-2 \geq x$$

$$x \leq -2$$

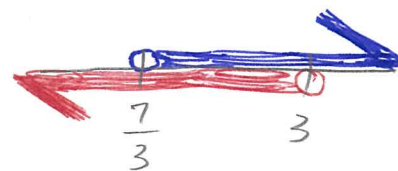


$$(-\infty, -2) \cup (2, \infty)$$

$$472) |3x-8| > -1$$

$$\begin{array}{lcl}
 +(3x-8) > -1 & \text{or} & \\
 3x-8 > -1 & \downarrow & \\
 \frac{+8}{+8} & \text{any color} & \\
 \hline
 3x > 7 & & \\
 \frac{3}{3} & & \frac{7}{3} \\
 x > \frac{7}{3} & & \\
 \frac{7}{3} < x & &
 \end{array}$$

$$\begin{array}{lcl}
 -(3x-8) > -1 & & \\
 -3x+8 > -1 & & \\
 \frac{+3x+1}{+1+3x} & & \\
 \hline
 9 > 3x & & \\
 \frac{9}{3} & & \frac{3x}{3} \\
 3 > x & & \\
 x < 3 & &
 \end{array}$$

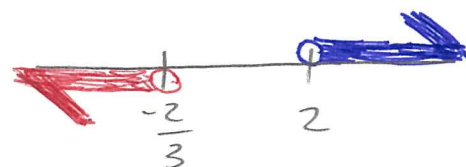

 $(-\infty, \infty)$

identity

$$474) |3x-2| > 4$$

$$\begin{array}{lcl}
 +(3x-2) > 4 & \text{or} & \\
 3x-2 > 4 & \downarrow & \\
 \frac{+2}{+2} & \text{any color} & \\
 \hline
 3x > 6 & & \\
 \frac{3}{3} & & \frac{6}{3} \\
 x > 2 & & \\
 2 < x & &
 \end{array}$$

$$\begin{array}{lcl}
 -(3x-2) > 4 & & \\
 -3x+2 > 4 & & \\
 \frac{+3x-4}{+3x-4-4+3x} & & \\
 \hline
 -2 > 3x & & \\
 \frac{-2}{3} & & \frac{3x}{3} \\
 -\frac{2}{3} > x & & \\
 x < -\frac{2}{3} & &
 \end{array}$$


 $(-\infty, -\frac{2}{3}) \cup (2, \infty)$

$$476) |x+3| \geq 5$$

$$\begin{array}{lcl}
 +(x+3) \geq 5 & \text{or} & \\
 x+3 \geq 5 & \downarrow & \\
 \frac{-3}{-3} & \text{any color} & \\
 \hline
 x \geq 2 & & \\
 2 \leq x & &
 \end{array}$$

$$\begin{array}{lcl}
 -(x+3) \geq 5 & & \\
 -x-3 \geq 5 & & \\
 \frac{+x-5}{+x-5-5+x} & & \\
 \hline
 -8 \geq x & & \\
 x \leq -8 & &
 \end{array}$$


 $(-\infty, -8] \cup [2, \infty)$

478) $3|x| + 4 \geq 1$

$$\begin{array}{r} -4 \quad -4 \\ \hline 3|x| \geq -3 \end{array}$$

$$\frac{3|x|}{3} \geq \frac{-3}{3}$$

$$|x| \geq -1$$

$$+(x) \geq -1$$

$$x \geq -1$$

$$-1 \leq x$$

or

$$-(x) \geq -1$$

$$-x \geq -1$$

$$\begin{array}{r} +1 \quad +x \quad +1 \quad +x \\ \hline 1 \geq x \end{array}$$

$$1 \geq x$$

$$x \leq 1$$

 $(-\infty, \infty)$ identity

480) $2|x+6| + 4 = 8$

$$\begin{array}{r} -4 \quad -4 \\ \hline 2|x+6| = 4 \end{array}$$

$$\frac{2|x+6|}{2} = \frac{4}{2}$$

$$|x+6| = 2$$

$$+(x+6) = 2$$

$$x+6 = 2$$

$$\begin{array}{r} -6 \quad -6 \\ \hline x = -4 \end{array}$$

$$x = -4$$

$$-(x+6) = 2$$

$$-x-6 = 2$$

$$\begin{array}{r} +x \quad -2 \quad -2 \quad +x \\ \hline -8 = x \end{array}$$

$$-8 = x$$

482) $|3x-4| \geq 2$

$$+(3x-4) \geq 2$$

or

$$3x-4 \geq 2$$

 $\downarrow$

$$\begin{array}{r} +4 \quad +4 \\ \hline 3x \geq 6 \end{array}$$

$$\frac{3x}{3} \geq \frac{6}{3}$$

$$x \geq 2$$

$$2 \leq x$$

any color

$$-(3x-4) \geq 2$$

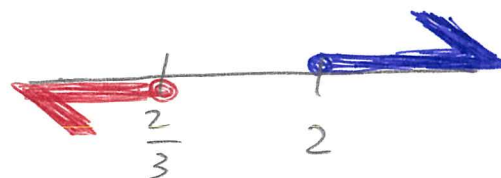
$$-3x+4 \geq 2$$

$$\begin{array}{r} +3x \quad -2 \quad -2 \quad +3x \\ \hline \frac{2}{3} \geq \frac{3x}{3} \end{array}$$

$$\frac{2}{3} \geq \frac{3x}{3}$$

$$\frac{2}{3} \geq x$$

$$x \leq \frac{2}{3}$$



$$(-\infty, \frac{2}{3}] \cup [2, \infty)$$

486) $|7x+2| + 8 < 4$

$$\begin{array}{r} -8 \quad -8 \\ \hline |7x+2| < -4 \end{array}$$

$$|7x+2| < -4$$

contradiction

no solution

$$484) |4x - 3| < 5$$

$$+(4x - 3) < 5 \quad \text{and}$$

$$4x - 3 < 5$$

$$\begin{array}{r} +3 \quad +3 \\ \hline 4x < 8 \\ \frac{4x}{4} < \frac{8}{4} \end{array}$$

$$x < 2$$

↓

"both colors"

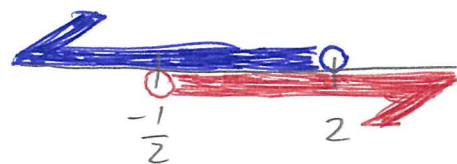
$$-(4x - 3) < 5$$

$$-4x + 3 < 5$$

$$+4x \quad -5 \quad -5 + 4x$$

$$\begin{array}{r} -2 < 4x \\ \frac{-2}{4} < \frac{4x}{4} \end{array}$$

$$-\frac{1}{2} < x$$



$$\left(-\frac{1}{2}, 2\right)$$

$$488) |8 - x| = |4 - 3x|$$

$$+(8 - x) = 4 - 3x$$

$$8 - x = 4 - 3x$$

$$\begin{array}{r} -8 + 3x \quad -8 + 3x \\ \hline \end{array}$$

$$\begin{array}{r} 2x = -4 \\ \frac{2x}{2} = \frac{-4}{2} \end{array}$$

$$x = -2$$

$$-(8 - x) = 4 - 3x$$

$$-8 + x = 4 - 3x$$

$$\begin{array}{r} +8 + 3x \quad +8 + 3x \\ \hline \end{array}$$

$$\begin{array}{r} 4x = 12 \\ \frac{4x}{4} = \frac{12}{4} \end{array}$$

$$x = 3$$