Answer each question neatly on the line provided.

Name: ______________________ ID: ______________

Quiz 1

1. (5 points) Solve the system $\begin{cases} 4x + 3y &= 2 \\ 7x + 5y &= 3 \end{cases}$

1. ________

2. (5 points) Solve the system $\begin{cases} x + y - z &= 0 \\ 4x - y + 5z &= 0 \\ 6x + y + 4z &= 0 \end{cases}$

2. ________

3. (5 points) Solve the system $\begin{cases} x - 2y &= 3 \\ 2x - 4y &= 8 \end{cases}$ Represent your solution graphically.

3. ________

Quiz 2

1. (5 points) For which values of k does the system $\begin{cases} x + 2y + 3z &= 0 \\ x + 3y + 8z &= 0 \\ x + 2y + 2z &= k \end{cases}$ have infinitely many solutions.

1. ______________

2. (5 points) Find all solutions of $\begin{cases} x + y &= 1 \\ 2x - y &= 5 \\ 3x + 4y &= 2 \end{cases}$

2. ______________

3. (5 points) Find all solutions of $\begin{cases} x_1 + 2x_3 + 4x_4 &= -8 \\ x_2 - 3x_3 - x_4 &= 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 &= 0 \\ -x_2 + 3x_3 + 4x_4 &= -12 \end{cases}$

3. ______________

Quiz 3

1. (5 points) (True/False) A linear system with more unknowns than equations must have infitely many solutions or none.

1. ______________

2. (5 points) (True/False) The rank of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ is 3.

2. ______________

3. (5 points) Describe the linear transformation $T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \vec{x}$ geometrically. (HINT: It may be helpful to draw the image of the "L" shape as is done in our textbook).

3. ______________

Quiz 4

1. (5 points) (True/False) A linear system with more unknowns than equations must have infitely many solutions or none.

1. ______________

2. (5 points) (True/False) The rank of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 5 \end{bmatrix}$ is 3.

2. ______________

3. (5 points) Describe the linear transformation $T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \vec{x}$ geometrically. (HINT: It may be helpful to draw the image of the "L" shape as is done in our textbook).

3. ______________

Quiz 5

1. (5 points) Find A^4 when $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

1. ______________

2. (5 points) Decide if the matrix $A = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is invertible. If it is find its inverse.

2. ______________

3. (5 points) (True/False) There is a matrix A so that $A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$.

3. ______________

Quiz 6

1. (5 points) If $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is in the span V of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, $\vec{v}_3 = \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix}$, find the coordinates of $\vec{x}$ with respect to the basis of V determined by $\vec{v}_1, \vec{v}_2, \vec{v}_3$. If $\vec{x}$ is not in V write, "not in V".

1. ________________

2. (5 points) For which value(s) of the constant k do the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$ form a basis of $\mathbb{R}^4$?

2. ________________

3. (5 points) Consider a linear transformation from T from $\mathbb{R}^5$ to $\mathbb{R}^3$. What are the possible values of nullity of T.

3. ________________

Quiz 7

1. (5 points) Find the matrix B of the linear transformation $T(x) = Ax = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ with respect to the basis $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

1. ______________

2. (5 points) Find a basis of the plane $\mathbb{R}^2$ so that the matrix B of the mirror reflection T of $\mathbb{R}^2$ over the line spanned by $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is a diagonal matrix.

2. ______________

3. (5 points) Find a basis for the space of all diagonal 2×2 matrices.

3. ______________

Quiz 8

1. (5 points) (True/False) The vectors $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}, \begin{bmatrix} 1 & 4 \\ 6 & 8 \end{bmatrix}$ are linearly independent.

1. ______________

2. (5 points) Find the matrix B of the linear transformation $T(f) = 3f' - 4f$ from P_2 to P_2 with respect to the basis $\mathfrak{U} = \left(1, t, t^2\right)$.

2. ______________

3. (5 points) Find the orthogonal projection of $\begin{bmatrix} 6 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ onto the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

3. ______________

Quiz 9

1. (5 points) Find the least squares solution to the system $Ax = b$ when $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} -6 \\ -6 \\ -6 \end{bmatrix}$.

1. ________________

2. (5 points) Find the best fit linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(0, -2), (1, -2), (1, -6)$ using least squares.

2. ________________

3. (5 points) Evaluate $\det A$ when $A = \begin{bmatrix} 10 & 20 & 30 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$.

3. ________________

Quiz 10

1. (5 points) Find an eigenvector of $A = \begin{bmatrix} -6 & 6 \\ -15 & 13 \end{bmatrix}$ with corresponding eigenvalue 4.

1. ______________

2. (5 points) Find the eigenvalue of $A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$ associated to the eigenvector $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$.

2. ______________

3. (5 points) Find the classical adjoint of $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

3. ______________

Quiz 11

1. (5 points) Find all real eigenvalues of $A = \begin{bmatrix} 4 & 5 \\ -2 & -2 \end{bmatrix}$ with their multiplicities

1. ________________

2. (5 points) Find all real eigenvalues of $A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 3 & 0 \\ 2 & 1 & 2 & 2 \end{bmatrix}$ with their multiplicities

2. ________________

3. (5 points) Find an eigenbasis of $A = \begin{bmatrix} 6 & 3 \\ 2 & 7 \end{bmatrix}$. Write NONE on the answerline if there is no eigenbasis.

3. ________________