Name: ____________________ ID: ____________

Answer 18 of the following 21 questions.

1. (5 points) Find a basis of the plane $x_1 + 2x_2 + x_3 = 0$ in $\mathbb{R}^3$.

2. (5 points) Find all solutions to

$$\begin{cases} x_1 + 2x_3 + 4x_4 = -8 \\ x_2 - 3x_3 - x_4 = 6 \\ 3x_1 + 4x_2 - 6x_3 + 8x_4 = 0 \\ -x_2 + 3x_3 + 4x_4 = -12 \end{cases}.$$

3. (5 points) (True/False) The rank of the matrix $A = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$ is two.

4. (5 points) (True/False) There exists a $4 \times 3$ matrix $A$ of rank 3 so that $A \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \vec{0}$.

5. (5 points) Geometrically interpret the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{x}$$

as a rotation, mirror reflection, or projection. (Be specific. If the linear transformation is a rotation, state the the center and angle. If the linear transformation is a projection or a mirror reflection, give an equation of the line.)

6. (5 points) Find the matrix $B$ of the linear transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix})$ when $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

7. (5 points) Compute the matrix product $\begin{bmatrix} 0 & 0 & 10 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$

8. (5 points) (True / False) Matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ is invertible.

9. (5 points) Find a basis for the kernel of $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \end{bmatrix}$

10. (5 points) Fit a linear function of the form $y = c_0 + c_1 t$ to the data points $(0,0), (1,1), (2,1)$ using least squares.

11. (5 points) Evaluate $\det \begin{bmatrix} 0 & 7 & 5 & 3 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 2 & -1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$.

12. (5 points) Find an orthonormal basis of the image of $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & -2 & 0 \end{bmatrix}$.

13. (5 points) Find an eigenbasis of the following linear transformation of $\mathbb{R}^2$ : projection onto the line $L$ spanned by $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

14. (5 points) Find an orthogonal matrix $S$ so that $S^T AS$ is diagonal when $A = \begin{bmatrix} 3 & 3 \\ 3 & -5 \end{bmatrix}$.

15. (5 points) Find the projection of $\vec{b} = \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}$ onto the image of $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$.

16. (5 points) Find the singular values of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$.

17. (5 points) Find the eigenvalues of $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.

18. (5 points) (True/False) The matrix $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ is diagonalizable.

19. (5 points) Find the coordinates of $\vec{x} = \begin{bmatrix} 7 \\ 16 \end{bmatrix}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 5 \\ 12 \end{bmatrix})$

20. (5 points) Find the rank of the linear transformation $T(f) = f' - 3f$ from $P_2$ to $P_2$.

21. (5 points) The matrix $\begin{bmatrix} k^2 & 1 & 4 \\ k & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible for all real numbers $k$.