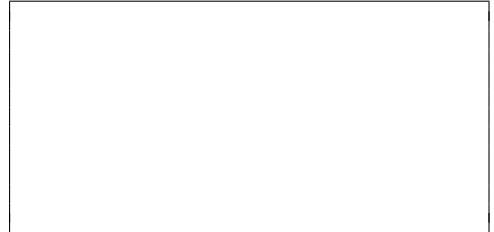


Name: \_\_\_\_\_

ID: \_\_\_\_\_

Answer 18 of the following 21 questions.

1. (5 points) Find all vectors  $\vec{x}$  such that  $A\vec{x} = \vec{b}$  when  $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  and  $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ .



2. (5 points) Find all solutions to

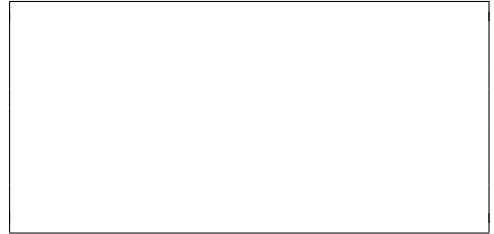
$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 4x_2 + 7x_3 = 2 \\ 3x_1 + 7x_2 + 11x_3 = 8 \end{cases}.$$



3. (5 points) We are told that  $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$  is an eigenvector of the matrix  $\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$ . What is its associated eigenvalue?



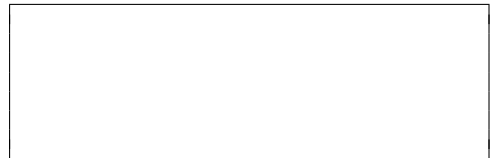
4. (5 points) Find an orthonormal eigenbasis of  $L$ , the reflection of  $\mathbb{R}^3$  about the line spanned by  $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ .



5. (5 points) Find the matrix  $B$  of the linear transformation  $T(\vec{x}) = A\vec{x}$  with respect to the basis  $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix})$  when  $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ .



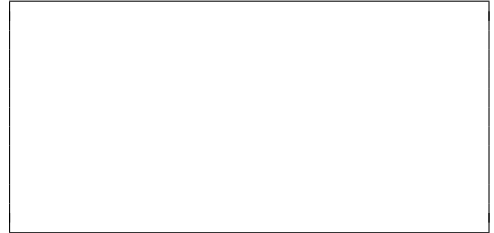
6. (5 points) Find the redundant column(s) of  $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ . If there are no redundant columns, write "none".




7. (5 points) Find a basis for the image of  $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$



8. (5 points) Find the orthogonal projection of  $\vec{x} = \begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$  onto the subspace spanned by  $\vec{v}_1 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$  and  $\vec{v}_2 = \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ .



9. (5 points) Find the determinant of the linear transformation  $T(f) = 2f + 3f'$  from  $P_2$  to  $P_2$ .



10. (5 points) Find an orthonormal basis of the image of the matrix  $A = \begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$ .



11. (5 points) Find a diagonal matrix  $B$  so that  $B = S^{-1}AS$  is diagonal when  $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$ .



12. (5 points) Give an example of a  $2 \times 2$  matrix  $A$  whose image is spanned by the vector  $\begin{bmatrix} 1 \\ -5 \end{bmatrix}$ .

13. (5 points) Find an orthogonal matrix  $S$  so that  $S^T A S$  is diagonal when  $A = \begin{bmatrix} 3 & 3 \\ 3 & -5 \end{bmatrix}$ .

14. (5 points) Find the matrix  $\Sigma$  in the singular value decomposition  $A = U \Sigma V^T$  of  $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ .

15. (5 points) Let  $V$  be the span of  $(\vec{v}_1 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix})$ . Find the coordinates of  $\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$  with respect to the basis  $\mathfrak{B} = (\vec{v}_1, \vec{v}_2)$ .

16. (5 points) (True/False): The matrix  $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  is diagonalizable.

17. (5 points) (True/False): If  $\vec{v}$  is an eigenvector of an  $n \times n$  matrix  $A$  then  $\vec{v}$  must be in the kernel of  $A$ .

18. (5 points) (True/False):  $x^* = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$  is the least squares solution of the system  $Ax = b$  when  $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$  and  $b = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$ .

19. (5 points) (True/False): If the determinant of a  $4 \times 4$  matrix  $A$  is 4 then the rank of  $A$  must be 4.

20. (5 points) (True/False):  $\det \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 10 & 0 & 0 & 0 \end{bmatrix} = -10$ .

21. (5 points) (True/False):  $(\operatorname{im} A)^\perp = \ker(A^T)$  when  $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ .