Name: ______________________ ID: ______________

Answer 18 of the following 21 questions.

1. (5 points) Find all vectors $\vec{x}$ such that $A\vec{x} = \vec{b}$ when $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

2. (5 points) Find all solutions to

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 1 \\ 2x_1 + 4x_2 + 7x_3 = 2 \\ 3x_1 + 7x_2 + 11x_3 = 8 \end{cases}.$$

3. (5 points) We are told that $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$. What is its associated eigenvalue?

4. (5 points) Find an orthonormal eigenbasis of L, the reflection of $\mathbb{R}^3$ about the line spanned by $\vec{v} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

5. (5 points) Find the matrix B of the linear transformation $T(\vec{x}) = A\vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix})$ when $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$.

6. (5 points) Find the redundant column(s) of $\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$. If there are no redundant columns, write "none".

7. (5 points) Find a basis for the image of $A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix}$

8. (5 points) Find the orthogonal projection of $\vec{x} = \begin{bmatrix} 9 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ onto the subspace spanned by $\vec{v}_1 \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\vec{v}_2 \begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

9. (5 points) Find the determinant of the linear transformation $T(f) = 2f + 3f'$ from P_2 to P_2.

10. (5 points) Find an orthonormal basis of the image of the matrix $A = \begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$.

11. (5 points) Find a diagonal matrix B so that $B = S^{-1}AS$ is diagonal when $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$.

12. (5 points) Give an example of a 2×2 matrix A whose image is spanned by the vector $\begin{bmatrix} 1 \\ -5 \end{bmatrix}$.

13. (5 points) Find an orthogonal matrix S so that $S^T AS$ is diagonal when $A = \begin{bmatrix} 3 & 3 \\ 3 & -5 \end{bmatrix}$.

14. (5 points) Find the matrix Σ in the singular value decomposition $A = U\Sigma V^T$ of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

15. (5 points) Let V be the span of $(\vec{v}_1 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix})$. Find the coordinates of $\vec{x} = \begin{bmatrix} 3 \\ -4 \end{bmatrix}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1, \vec{v}_2)$.

16. (5 points) (True/False): The matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable.

17. (5 points) (True/False): If $\vec{v}$ is an eigenvector of an $n \times n$ matrix A then $\vec{v}$ must be in the kernel of A.

18. (5 points) (True/False): $x^* = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ is the least squares solution of the system $Ax = b$ when $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$.

19. (5 points) (True/False): If the determinant of a 4×4 matrix A is 4 then the rank of A must be 4.

20. (5 points) (True/False): $\det \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 10 & 0 & 0 & 0 \end{bmatrix} = -10.$

21. (5 points) (True/False): $(\mathrm{im} A)^{\perp} = \ker(A^T)$ when $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$.