Name: ______________________________ ID: ____________________

Answer 18 of the following 20 questions.

1. (5 points) Find all solutions to

$$\begin{cases} x_3 + x_4 = 0 \\ x_2 + x_3 = 0 \\ x_1 + x_2 = 0 \\ x_1 + x_4 = 0 \end{cases}. \tag{1}$$

1. ______________

2. (5 points) (True/False) There exists a 4×3 matrix A of rank 1 so that $A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \vec{0}$.

2. ______________

3. (5 points) Interpret the linear transformation

$$T(\vec{x}) = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix} \vec{x}$$

geometrically. State the angle and ceneter of rotations and write an equation of the line for any mirror reflection or orthogonal projection.

4. (5 points) Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

4. ______________

5. (5 points) Find which value(s) of the constant k do the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$ form a basis of $\mathbb{R}^4$.

5. ______________

6. (5 points) Find the matrix B of the linear transformation $T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix})$.

6. ______________

7. (5 points) (TRUE/FALSE) $T(f(t)) = \int_{-2}^{3} f(t)dt$ is an isomorphism from P_2 to $\mathbb{R}$.

7. ______________

8. (5 points) Find an orthonormal basis of the plane $x_1 + x_2 + x_3 = 0$.

8. ______________

9. (5 points) Find a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(0,0), (0,1), (1,1)$ using least squares.

9. ______________

10. (5 points) Evaluate $\det \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 0 \end{bmatrix}$.

10. ______________

11. (5 points) (TRUE/FALSE) If the determinant of a 4×4 matrix is 0 then its rank must be 0.

11. ______________

12. (5 points) Diagonalize $A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$.

12. ______________

13. (5 points) Find an orthonormal eigenbasis for $A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$.

13. ______________

14. (5 points) Find the definiteness of the quadratic form $q(x_1, x_2) = x_1^2 + 4x_1x_2 + x_2^2$.

14. ______________

15. (5 points) Find the singular values of $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

15. ______________

16. (5 points) (True/False) There exists a matrix A so that $A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$.

16. ______________

17. (5 points) Consider the linear transformation T from $\mathbb{R}^3$ to $\mathbb{R}^4$ and some linearly dependent set of vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ of $\mathbb{R}^3$. Are the vectors $T(\vec{v}_1), T(\vec{v}_2), T(\vec{v}_3)$ necessarily dependent? Explain.

18. (5 points) (True/False) The column vectors of a 5×4 matrix A must be linearly dependent.

19. (5 points) Consider the vectors $\vec{u}_1 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$, $\vec{u}_2 = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$, $\vec{u}_3 = \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$. Find a vector $\vec{u}_4$ so that $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4$ are orthonormal.

19. ______________

20. (5 points) The matrix $\begin{bmatrix} k^2 & 1 & 4 \\ k & -1 & -2 \\ 1 & 1 & 1 \end{bmatrix}$ is invertible for all real numbers k.

20. ______________