Math 346 Draft Final Exam

Name: _____

1. (10 points) Find all solutions to
$$\begin{cases} x_3 + x_4 = 0 \\ x_2 + x_3 = 0 \\ x_1 + x_2 = 0 \\ x_1 + x_4 = 0 \end{cases}. \tag{1}$$

1. _____

2. (5 points) (True/False) There exists a 4×3 matrix A of rank 3 so that $A \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \vec{0}$.

2. _____

3. (5 points) Interpret the linear transformation
$$T(\vec{x}) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \vec{x}$$
geometrically.

4. (10 points) Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$.

4. _____

5. (10 points) Find which value(s) of the constant k do the vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ k \end{bmatrix}$ form a basis of $\mathbb{R}^4$.

5. _____

6. (10 points) Find the matrix B of the linear transformation $T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix})$.

6. _____

7. (5 points) (TRUE/FALSE) $T(f(t)) = \int_{-2}^{3} f(t)dt$ is an isomorphism from P_2 to $\mathbb{R}$.

7. _____

8. (10 points) Find an orthonormal basis of the plane $x_1 + x_2 + x_3 = 0$.

8. _____

9. (10 points) Find a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(0,0), (0,1), (1,1)$ using least squares.

9. _____

10. (5 points) Evaluate $\det \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 0 \end{bmatrix}$.

10. _____

11. (5 points) (TRUE/FALSE) If the determinant of a 4×4 matrix is 4 then its rank must be 4.

11. _____

12. (10 points) Diagonalize $A = \begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}$.

12. _____

13. (5 points) Find an orthonormal eigenbasis for $A = \begin{bmatrix} 6 & 2 \\ 2 & 3 \end{bmatrix}$.

13. _____

14. (5 points) Find the definiteness of the quadratic form $q(x_1, x_2) = x_1^2 + 4x_1 x_2 + x_2^2$.

14. _____

15. (5 points) Find the singular values of $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

15. _____