Sample Final B

Name: _____   EMPLID: _____

1. (5 points) Evaluate $\int 5^x \tan(5^x)\, dx$

2. (5 points) Evaluate $\frac{d}{dx}\left(\sqrt{\log_3 x}\right)$

3. (5 points) Evaluate $\int_{-\pi}^{\pi} \sin(x^3)\, dx$.

4. (5 points) Let $y(x)$ be the solution to the differential equation $y' = \frac{12x^3 + 12x^2}{y^2 e^{y^3}}$ for which $y(1) = 0$. Find $y(x)$ explicitly as a function of x.

5. (5 points) Evaluate $\int 4x \cos(x^2 + 2)\, dx$.

6. (5 points) Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings. A bottle of iced tea at room temperature ($72°$F) is placed in a refrigerator where the temperature is $44°$F. After half an hour the tea has cooled to $61°$F. How long does it take for the tea to cool to $50°$F?

7. (5 points) Evaluate $\int \sec(3x + 4)\, dx$.

8. (5 points) Evaluate $\int_0^1 \arctan x\, dx$.

9. (5 points) Evaluate $\int_0^2 \frac{2x^2-2x-4}{x^3+2x^2} \, dx$.

10. (5 points) Evaluate $\int (\tan x + \sin^2 x \cos x) \sin 2x \, dx$.

11. (5 points) Evaluate $\int \sqrt{1 - 4t^2} \, dt$.

12. (5 points) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{n(x+1)^n}{5^n \sqrt{n^2+4}}$.

13. (5 points) Use the Trapezoidal Rule with $n = 4$ to approximate $\int_0^2 \sqrt{1 + x^3} \, dx$. Leave your answer as an unsimplified sum.

14. (5 points) Use power series to approximate $\int_0^{0.3} \frac{x}{1+x^3} dx$ to six decimal places. Leave your answer as an unsimplified sum. Explain how your approximate is correct to six decimal places.

15. (10 points) State, for each series, whether it converges absolutely, coverges conditionally or diverges. Name the test used to support each conclusion, and show the work to apply the test.

    (a) $\sum_{n=1}^{\infty}(-1)^n(1 + \frac{1}{n} + \frac{1}{n^2})^n$

    (b) $\sum_{n=0}^{\infty} \frac{n^2 3^n \ln(n+2)}{2^{2n+2}}$

    (c) $\sum_{n=0}^{\infty} \frac{(-1)^n(n^2+3)}{n^3+4}$

16. (10 points) Which of the following improper integrals are convergent and which are divergent? Show why. Evaluate one of them.

(a) $\int_0^{\frac{\pi}{2}} \tan^2 \theta \, d\theta$

(b) $\int_1^\infty \frac{1}{x^2+4} \, dx$

(c) $\int_1^\infty \frac{x \arctan x}{\sqrt[3]{x^7+7}} \, dx$

17. (5 points) Sketch the graph of the polar equation $r = 3 + 2\sin\theta$ and find the area that is both inside the graph and above the x−axis.

18. (5 points) Graph $4x^2 + 36y^2 + 9z^2 + 16x - 20 = 0$ and sketch its trace in the xy−plane.