The City College Department of Mathematics
Fall 2025 Math 20212 Practice Final Exam

NAME: _____

YOUR INSTRUCTOR: _____

CALCULATORS are NOT allowed.
Please show all your work. No credit will be given for answers without clear and complete solutions.
Use only the paper provided. You may write on the back if you need more space, but clearly indicate this on the front.
There are 10 problems for a total of 100 points.

Problem 1		Problem 6			
Problem 2		Problem 7			
Problem 3		Problem 8			
Problem 4		Problem 9			
Problem 5		Problem 10		**Total**	

1. Compute each of the following integrals (6 points each).

a. $\displaystyle\int_0^1 \frac{x^2+x+1}{(x+1)^2(x+2)}\,dx$

b. $\displaystyle\int \sin^2 x \sin 2x\,dx$

c. $\displaystyle\int \frac{\sqrt{x^2-1}}{x^4}\,dx$

d. $\displaystyle\int (\arcsin x)^2\,dx$

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2. Evaluate the limits (4 points each).

a. $\lim_{n\to\infty} \dfrac{(-1)^n \cos(\pi n)}{\sqrt{n}}$

b. $\lim_{n\to\infty} n^2 e^{-n}$

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3. State for each series whether it converges absolutely, converges conditionally, or diverges. Name a test which supports your conclusion and justify why it applies, by showing a calculation or giving an explanation. (4 points each).

(a) $\displaystyle\sum_{n=3}^{\infty} \frac{\ln n}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n^2-1}}$

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4. Evaluate each of the following integrals or show that it is divergent (5 points each).

a. $\displaystyle\int_0^\infty \frac{x}{(x^2+1)^3}\,dx$

b. $\displaystyle\int_0^{\pi/2} \frac{\cos x}{\sqrt{\sin x}}\,dx.$

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5. (5 points each)

a. Without attempting to evaluate it, determine whether the following integral converges or diverges. Justify your answer.

$$\int_1^\infty \frac{\arctan x}{\sqrt{e^x + 2}}\, dx$$

b. Write out the form of the partial fraction decomposition of the following function. Do not attempt to determine the numerical values of the coefficients.

$$f(x) = \frac{12x - 5}{(x-1)(x+2)^2(x^2+x+3)^2}$$

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6. (6 points) Find the radius of convergence and determine the exact interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$.

7. For $f(x) = \arctan(2x)$

a. find the first three terms of its Taylor series centered at $a = \dfrac{\sqrt{3}}{2}$;

b. use the geometric series to find its Taylor series centered at $a = 0$.

(4 points each)

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8. (6 points) Draw a sketch of the conic whose equation is

$$x^2 + 4y^2 - 6x + 5 = 0.$$

Identify which sort of conic it is. On your sketch, show and label whichever of the following are present: vertices, asymptotes, and foci.

9.(5 points each)

a. A radioactive substance loses half of its mass every 8 years. Assuming exponential decay, after how many years will the substance be reduced to one-third of its original mass?

b. Find the solution of the differential equation which satisfies the given initial condition

$$x + 3y^2\sqrt{x^2+1}\frac{dy}{dx} = 0, \quad y(0) = 1$$

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10. (6 points) Sketch the curve given by the equation $r = 3 - 2\sin\theta$ in polar coordinates, labeling the x and y intercepts, and compute the area it encloses.
