Sample Final A

Name: _____   EMPLID: _____

1. (5 points) Find explicitly the function $y(x)$ that satisfies the differential equation $\frac{dy}{dx} = y^2 \sin x$ and the initial condition $y(0) = -1$.

2. (5 points) An 80 gram sample of radioactive material with a half-life of 2 days is brought into a lab. Find the function $N(t)$ of the number of grams of the material that remains t days after being received by the lab.

3. (5 points) Evaluate $\int \tan(3x)\,dx$.

4. (5 points) Evaluate $\int \frac{1}{\sqrt{1+x^2}}\,dx$.

5. (5 points) Evaluate $\int_0^{\frac{\pi}{2}} (x + \pi) \sin x \, dx$.

6. (5 points) Evaluate $\int (e^x \sin(e^x) + 4\cos^2(2x)) \, dx$.

7. (5 points) Evaluate $\int \left(\frac{1}{1+x^2} + \cos^3 x \sin^3 x \right) \, dx$.

8. (5 points) Evaluate $\int 5^{-x} \, dx$.

9. (5 points) Evaluate $\int \frac{2x^2+x+1}{x^3+x^2} \, dx$.

10. (5 points) Evaluate $\int_0^{\frac{\pi}{8}} \sec^2(2x) \, dx$.

11. (5 points) Find $\lim_{n\to\infty} \frac{\ln n}{1+\sqrt{n}}$.

12. (5 points) Find the interval of convergence of $\sum_{n=0}^{\infty} \frac{2^n (x+2)^n}{n+1}$.

13. (10 points) Show why each of the following series converges or diverges, and find the numerical value of one of them.

(a) $\sum_{n=2}^{\infty} \left(\frac{4}{n^2} - \frac{4}{(n+1)^2} \right)$.

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n 4^{n+1}}{5^n}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^n 7^{n+1}}{2^{2n-1}}$

14. (5 points) Let $f(x) = e^{x^3}$.

    (a) Find the first three nonzero terms of the Maclaurin series of f.

    (b) Use the answer to (a) to approximate $f'(\frac{-1}{2})$ with error less than .01. Show that your estimate has the required accuracy.

15. (10 points) State, for each series, whether it converges absolutely, coverges conditionally or diverges. Name the test used to support each conclusion, and show the work to apply the test.

    (a) $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n^3+1}$

    (b) $\sum_{n=1}^{\infty} (-1)^n (1 - \frac{1}{n^2})$

    (c) $\sum_{n=1}^{\infty} \frac{(-1)^n n}{2^n}$

16. (5 points) Which of the following improper integrals are convergent and which are divergent? Show why. Evaluate one of them.

    (a) $\int_4^\infty \frac{x^2\sqrt{x}}{x^3-x-1}\,dx$

    (b) $\int_1^\infty xe^{-2x}\,dx$

17. (10 points) (a) Graph $r = 4 + \cos\theta$ and $r = 1$ on the same rectangular coordinates, and label all $x-$ and $y-$ intercepts.

    (b) Find the area between the two graphs and write your answer in the answerbox.