346 Sample Final D

Name: _____ EMPLID: _____

1. (5 points) (TRUE/FALSE): There exists a 5×4 matrix whose image consists of all $\mathbb{R}^5$. How can you tell? (No credit will be given without an explanation.)

1. _____

2. (5 points) Find all real numbers k so that the system

$$\begin{cases} x_1 + x_2 - x_3 = 2 \\ x_1 + 2x_2 + x_3 = 3 \\ x_1 + x_2 + (k^2 - 5)x_3 = 4 \end{cases}.$$

has a unique solution.

3. (5 points) Find the matrix B of the linear transformation $T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \vec{x}$ with respect to the basis $\mathcal{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix})$.

4. (5 points) Find an eigenbasis of $A = \begin{bmatrix} 0.6 & 0.8 \\ 0.8 & -0.6 \end{bmatrix}$, representing the reflection about the line $L = \text{span} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

5. (5 points) Find the rank and nullity of the linear transformation $T(f) = f + 2f'$ from P_2 to P_2.

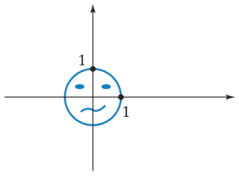
6. (5 points) Find an orthogonal matrix S and a diagonal matrix D such that $S^{-1}AS = D$ when $A = \begin{bmatrix} 3 & 3 \\ 3 & -5 \end{bmatrix}$.

7. (5 points) (TRUE/FALSE) $\vec{x} = \begin{bmatrix} -2 \\ 1 \\ -3 \\ 1 \\ 0 \end{bmatrix}$ is in the kernel of $A = \begin{bmatrix} 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. How can you tell? (No credit will be given without an explanation.)

7. _____

8. (5 points) Interpret the linear transformation

$$T(\vec{x}) = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix} \vec{x}$$

geometrically by sketching the effect of T on this 

circular face. Draw your face on top of answerline.

8. _____

9. (5 points) Describe the image of A^2 geometrically when $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$.

10. (5 points) Find the QR factorization of $A = \begin{bmatrix} 1 & 1 \\ 1 & 9 \\ 1 & -5 \\ 1 & 3 \end{bmatrix}$.

11. (5 points) Two interacting populations of h and f can be modeled by the recursive equations

$$\begin{cases} h(t+1) = 6h(t) + 3f(t) \\ f(t+1) = 2h(t) + 7f(t) \end{cases}.$$

Find $f(10)$ if $h(0) = 2, f(0) = 2$. You may leave exponents in your answer.

12. (5 points) Find the orthogonal projection of $\vec{x} = 10\vec{e}_1$ onto the the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$.

13. (5 points) Fit a linear function of the form $f(t) = c_0 + c_1 t$ to the data points $(0,0), (1,1), (1,2), (2,0)$ using least squares.

14. (5 points) Let V be the span of ($\vec{v}_1 = \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -10 \\ 0 \end{bmatrix}$). Find the coordinates of $\vec{x} = \begin{bmatrix} 20 \\ 3 \end{bmatrix}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1, \vec{v}_2)$.

15. (5 points) Find the matrix V in the singular value decomposition $A = U\Sigma V^T$ of $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$.

16. (5 points) (TRUE/FALSE): The matrix $\begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ is diagonalizable. How can you tell? (No credit will be given without an explanation.)

16. _____

17. (5 points) (TRUE/FALSE): The quadratic form $q(x_1, x_2) = x_1^2 + \frac{1}{2}x_1x_2 + x_2^2$ is positive definite. How can you tell? (No credit will be given without an explanation.)

17. _____

18. (5 points) (TRUE/FALSE): If a nonzero vector $\vec{v}$ is in the kernel of an $n \times n$ matrix A then $\vec{v}$ must be an eigenvector of A. How can you tell? (No credit will be given without an explanation.)

18. _____

19. (5 points) (TRUE/FALSE): $b - Ax^*$ is perpendicular to the image of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix}$ when x^* is the least squares solution of the system $Ax = b$ for $b = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$. How can you tell? (No credit will be given without an explanation.)

19. _____

20. (5 points) (TRUE/FALSE): If all the entries of a 9×9 matrix A are 9 then A is invertible. How can you tell? (No credit will be given without an explanation.)

20. _____