You have 1hr 15min. Answer each non-graph question neatly on the line provided.

Name: ____

1. (10 points) Is the vector $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ is in the span of the vectors $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \vec{v}_2 \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}$? If $\vec{x}$ is in the span $\vec{v}_1, \vec{v}_{2,3}$, write the coordinates of $\vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1, \vec{v}_2, \vec{v}_3)$ on the answerline. If $\vec{x}$ is not in the span $\vec{v}_1, \vec{v}_{2,3}$ write FALSE on the answer line.

1. ________________

2. (a) (5 points) (TRUE/FALSE) The set W of all noninvertible 2×2 matrices is a subsapce of $\mathbb{R}^{2\times 2}$.

   (a) ________________

   (b) (5 points) (TRUE/FALSE) The set V of all invertible 2×2 matrices is a subsapce of $\mathbb{R}^{2\times 2}$.

   (b) ________________

3. (10 points) Find a basis of all polynomials $f(t)$ in P_2 such that $f(1) = 0$.

4. (10 points) Find the image, rank, kernel, and nullity of the transformation $T(M) = M \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ from $\mathbb{R}^{2\times 2}$ to $\mathbb{R}^{2\times 2}$.

5. (10 points) Find the matrix of the linear transformation $T(f) = f'' + 4f'$ from P_2 to P_2 with respect to the basis $\mathfrak{U} = (1, t, t^2)$ of P_2.

5. ________________

6. Find the rank and nullity of the linear transformation $T(f(t) = f''(t) + 4f(t)$ from P_2 to P_2.

6. ________________

7. (10 points) Find the orthogonal projection of $9\vec{e}_1$ onto the subspace of $\mathbb{R}^4$ spanned by $\begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 2 \\ 0 \\ 1 \end{bmatrix}$.

7. ________________

8. (10 points) Perform the Gram-Schmidt process on the sequence of vectors $\vec{v}_1 = \begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 25 \\ 0 \\ -25 \end{bmatrix}$.

8. ________________

9. (10 points) Determine the error $\|\vec{b} - A\vec{x}^*\|$ when $\vec{x}^*$ is the least squares solution of the system $A\vec{x} = \vec{b}$ where $A = \begin{bmatrix} 6 & 9 \\ 3 & 8 \\ 2 & 10 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 0 \\ 49 \\ 0 \end{bmatrix}$.

9. ______________

10. (10 points) Find the determinant of $\begin{bmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{bmatrix}$

10. ______________

11. (a) (5 points) (TRUE/FALSE) The determinant of any diagonal $n \times n$ matrix is the product of the diagonal entries.

(a) ______________

(b) (5 points) (TRUE/FALSE) $\det(A + B) = \det(A) + \det(B)$ for all 5×5 matrices A and B.

(b) ______________