

Name: _____

1. (10 points) Find an orthonormal basis of the plane $x_1 + 2x_2 = 0$.

1. _____

2. (10 points) Find the least squares solution $\vec{x}^*$ of $A\vec{x} = \vec{b}$ when $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

2. _____

3. (10 points) Find the best fit line $y = mx + b$ to the data points $(-1, 1), (1, 1), (2, 3)$ using least squares.

3. _____

4. (10 points) For which values of t is $\begin{bmatrix} t & 1 & 0 \\ 2 & t & 2 \\ 0 & 1 & t \end{bmatrix}$ invertible?

4. _____

5. (10 points) Find the QR factorization of the matrix $\begin{bmatrix} 4 & 25 & 0 \\ 0 & 0 & -2 \\ 3 & -25 & 0 \end{bmatrix}$.

5. _____

6. (10 points) (True/False) All $n \times n$ orthogonal matrices Q are invertible. Justify your answer for full credit.

6. _____

7. (10 points) Find the matrix B of the linear transformation $T(\vec{x}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix})$.

7. _____
8. (10 points) (True/False) $\begin{bmatrix} -10 \\ 10 \\ 10 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 4 & 1 & 1 \\ -5 & 0 & -3 \\ -1 & -1 & 2 \end{bmatrix}$. Justify your answer for full credit.

8. _____
9. (10 points) Find the image, rank, kernel, and nullity of the transformation $T(f(t)) = t(f'(t))$ from P_2 to P_2 .

9. _____

10. (10 points) Determine whether $\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$ is in the span V of $\vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 0 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} 0 \\ 0 \\ 4 \\ 1 \end{bmatrix}$. If $\vec{x}$ is in V find the coordinates of $\vec{x}$ with respect to the basis $\mathfrak{B} = (\vec{v}_1, \vec{v}_2, \vec{v}_3)$.

10. _____

11. (10 points) Consider the orthonormal vectors $\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4$, and $\vec{u}_5$ in $\mathbb{R}^{10}$. Find the length of the vector $\vec{x} = 7\vec{u}_1 - 3\vec{u}_2 + 2\vec{u}_3 + \vec{u}_4 - \vec{u}_5$.

11. _____