

You have 1hr 15min. Answer each non-graph question neatly on the line provided.

Name: _____

1. (10 points) For which value(s) of k does the system

$$\begin{cases} x_1 + x_2 - x_3 = -2 \\ 3x_1 - 5x_2 + 13x_3 = 18 \\ x_1 - 2x_2 + 5x_3 = k \end{cases} \quad (1)$$

have no solution.

1. _____

2. (10 points) (True/False) There is a sequence of elementary row operations that transforms the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \text{ into } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ?$$

2. _____

3. (10 points) (True/False) Vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ is a linear combination of the vectors $\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$.

3. _____

4. (10 points) Let A be a 4×4 matrix, and let \vec{b} and \vec{c} be two vectors in \mathbb{R}^4 . We are told that the system $A\vec{x} = \vec{b}$ is inconsistent. What can you say about the number of solutions of the system $A\vec{x} = \vec{c}$.

5. (10 points) Draw a sketch showing the effect of the linear transformation $T(\vec{x}) = A\vec{x}$ on the unit circle $x^2 + y^2 = 1$ when $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$.

6. (10 points) Find the matrix P of the orthogonal projection onto the line spanned by $\vec{v} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

6. _____

7. (10 points) Find the matrix of the mirror reflection about the $x - z$ plane in \mathbb{R}^3 .

7. _____

8. (10 points) (True/False) $\begin{bmatrix} 11 & 13 & 15 \\ 17 & 19 & 21 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 13 \\ 19 \\ 21 \end{bmatrix}$

8. _____

9. (10 points) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 3 & 9 \end{bmatrix}$. Find a basis of the image of A .

9. _____

10. (10 points) Find a basis of the subspace of \mathbb{R}^3 defined by $2x_1 + 3x_2 + x_3 = 0$

11. (10 points) (True/False) The image of a 3×4 matrix is a subspace of \mathbb{R}^4 .