

Name: _____

1. (10 points) For which values of c and d is $\begin{bmatrix} 5 \\ 7 \\ c \\ d \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$?

2. (10 points) Find a basis of the kernel and the image of $A = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 3 & 4 & 0 & 0 \\ 1 & 2 & -2 & -4 \end{bmatrix}$.
1. _____

3. (10 points) (True/False) If A is a 3×4 matrix of rank 3 then the system $A\vec{x} = \begin{bmatrix} 11 \\ -2 \\ 3 \end{bmatrix}$ must have infinitely many solutions. Justify your answer for full credit.
2. _____

3. _____

4. (10 points) Evaluate $\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^9$.

4. _____

5. (10 points) Find the inverse of the matrix $B = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

5. _____

6. (10 points) (True/False) Vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 6 \\ 7 \\ 8 \end{bmatrix}$, $\begin{bmatrix} 9 \\ 8 \\ 7 \\ 6 \end{bmatrix}$, $\begin{bmatrix} 5 \\ 4 \\ 3 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix}$ are linearly independent. Justify your answer for full credit.

6. _____

7. (10 points) (True/False): The column vectors of a 4×5 matrix must be linearly dependent. Justify your answer for full credit.

7. _____

8. (a) (5 points) Find the rotation matrix C that transforms $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ into $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$.

(a) _____

- (b) (5 points) Find the orthogonal projection matrix B that transforms $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ into $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$.

(b) _____

9. (10 points) Find all solutions of the following system of equations

$$\begin{cases} x_1 + 2x_2 + 2x_4 + 3x_5 = 0 \\ x_3 + 3x_4 + 2x_5 = 0 \\ x_3 + 4x_4 - x_5 = 0 \\ x_5 = 0 \end{cases}$$

9. _____

10. (10 points) Find a basis of the subspace of $\mathbb{R}^4$ that consists of all vectors perpendicular to both

$$\begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}$$

10. _____

11. (5 points) (True/False) There exists a 6×5 matrix whose image consists of all of $\mathbb{R}^6$. Justify your answer for full credit.

11. _____