Math 346 Midterm 1

Name: _____ ID: _____

1. (5 points) Solve the system $\begin{cases} 2x_1 + 4x_2 + 6x_3 &= 0 \\ 4x_1 + 5x_2 + 6x_3 &= 3 \\ 7x_1 + 8x_2 + 9x_3 &= 6 \end{cases}$

1. _____

2. (5 points) Find a redundant column of $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ and write it as a linear combination of preceding columns.

2. _____

3. (5 points) Find all solutions of the system $\begin{cases} x + 2y &= a \\ 3x + 5y &= b \end{cases}$ for arbitrary constants a and b.

3. _____

4. (5 points) Find all the vectors in $\mathbb{R}^3$ perpendicular to $\begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}$.

4. _____

5. (5 points) For which value(s) of c is $\begin{bmatrix} 3 \\ 5 \\ c \\ 9 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$?

5. _____

6. (5 points) (True/False) The rank of the matrix $\begin{bmatrix} -1 & -1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ is 1.

6. _____

7. (5 points) Find the inverse of the linear transformation $\begin{cases} y_1 &= x_1 + 7x_2 \\ y_2 &= 3x_1 + 20x_2. \end{cases}$

7. _____

8. (5 points) Find vectors that span the image of $A = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$. Give as few vectors as possible.

8. _____

9. (5 points) Describe $T(\vec{x}) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ geometrically in detail. (HINT: Sketch the image of the standard L shape under T.)

9. _____

10. (5 points) Let L be the line in $\mathbb{R}^3$ that consists of all scalar multiples of the vector $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$. Find the orthogonal projection of the vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ onto L.

10. _____

11. (5 points) Find vectors that span the kernel of $\begin{bmatrix} 2 & 3 \\ 6 & 9 \end{bmatrix}$.

11. _____

12. (5 points) Evaluate $\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$.

12. _____