

Name: _____

EMPLID: _____

1. (5 points) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=2}^{\infty} \frac{(x-2)^n}{2^n \ln n}$.

1. _____

2. (5 points) Current $I(t)$ is modeled in a simple circuit by the differential equation $L \frac{dI}{dt} + RI(t) = E(t)$. Find an expression for the current if the resistance $R = 12\Omega$, the inductance $L = 4\text{H}$, a battery gives a constant voltage $E(t) = 60\text{V}$, and the switch is turned on when $t = 0$, i.e. $I(0) = 0$.

2. _____

3. (5 points) Use a power series to approximate $\int_0^{0.3} \frac{x}{1+x^3} dx$ to four decimal places. You need not simplify your answer but you must justify your answer.

3. _____

4. (5 points) Find the first three nonzero terms of the Taylor series of $f(x) = \sqrt[3]{x}$ centered at $a = 8$.

4. _____

5. (5 points) Use series to evaluate $\lim_{x \rightarrow 0} \frac{x - \ln(1+x)}{x^2}$. Show your work. No credit will be given for any other method.

5. _____

6. (5 points) Evaluate $\int \frac{\tan^{-1} x}{x} dx$ as a power series. What is its radius of convergence?

6. _____

7. (5 points) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} n!(2x - 1)^n$.

7. _____

8. (5 points) (True/False) The series $\sum_{n=1}^{\infty} (-1)^n n e^{-n^2}$ converges absolutely.

8. _____

9. (5 points) Given two series

1. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

2. $\sum_{n=1}^{\infty} \left(\frac{n}{n^2+1}\right)^n$.

State which of the following are true.

- A. Series 1. is convergent and series 2. is convergent.
- B. Series 1. is divergent and series 2. is convergent.
- C. Series 1. is divergent and series 2. is divergent.
- D. Series 1. is convergent and series 2. is divergent.

9. _____

10. (5 points) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{nx^n}{5^n}$.

10. _____

11. (5 points) Find the Maclaurin series for $f(x) = x \sin x - x^2$.

11. _____

12. (5 points) Sketch the graph of the polar equation $r = 1 - \cos \theta$.