Math 20300
Calculus III

Lesson 20
Optimization

Dr. A. Marchese, The City College of New York

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Optimization Problems

(word problems to maximize or minimize)

Optimization problems (maximize or minimize):

1) read problem
2) read problem again
3) draw a picture (if possible)
4) assign variables to the quantities we must find
5) find relationships (equations) between the variables
6) Describe the function to be maximized or minimized in only two variables
7) solve
8) check
9) reread the problem and answer in the original language.

Ex. Find the point(s) on the surface $x^2 - yz = 5$ closest to the origin.

5) relationship between variables: $x^2 - yz = 5$

6) want to minimize $d = \sqrt{(x-0)^2 + (y-0)^2 + (z-0)^2}$

$$= \sqrt{x^2 + y^2 + z^2}$$

easier to minimize $d^2 = x^2 + y^2 + z^2$

notice $x^2 = 5 + yz$ $\quad d^2 = 5 + yz + y^2 + z^2 = f(y,z)$

minimize $f(y,z) = 5 + yz + y^2 + z^2$

$f_y(y,z) = z + 2y = 0 \qquad f_z(y,z) = y + 2z = 0$

$z = -2y \longrightarrow +y + 2(-2y) = 0$

$-3y = 0$

$y = 0$

$\therefore z = 0$

critical point at $\underset{(y,z)}{(0,0)}$

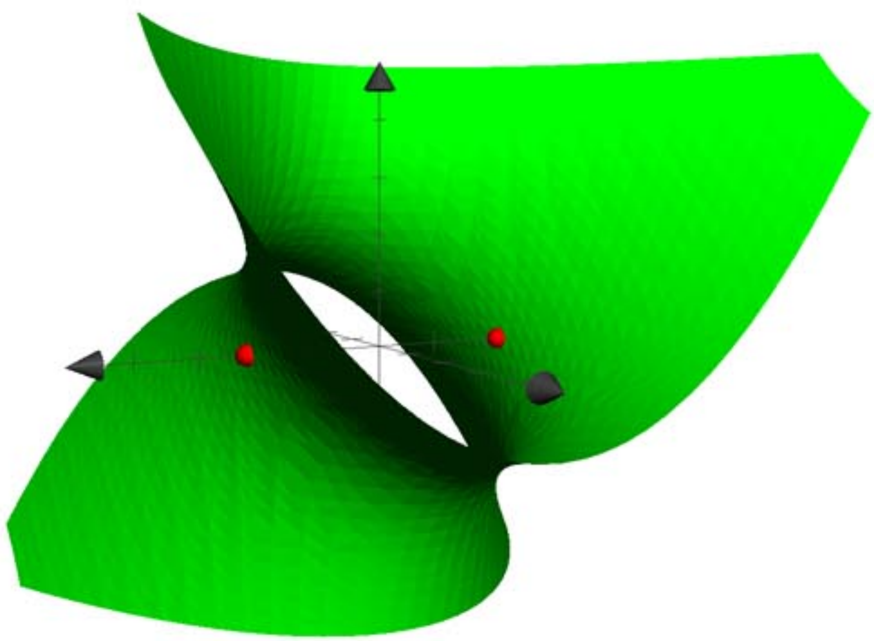
$f_{yy}(y,z) = 2 \qquad f_{zz}(y,z) = 2 \qquad f_{yz}(y,z) = 1$

$D = 2 \cdot 2 - (1)^2 > 0 \quad f_{yy}(0,0) > 0 \Rightarrow$ local <u>min</u>.

Since the domain of f is $\mathbb{R}^2$, and this is the only critical point, must be the absolute min as well.

So we find x: $x^2 = 5 + yz = 5 \quad x = \pm\sqrt{5}$

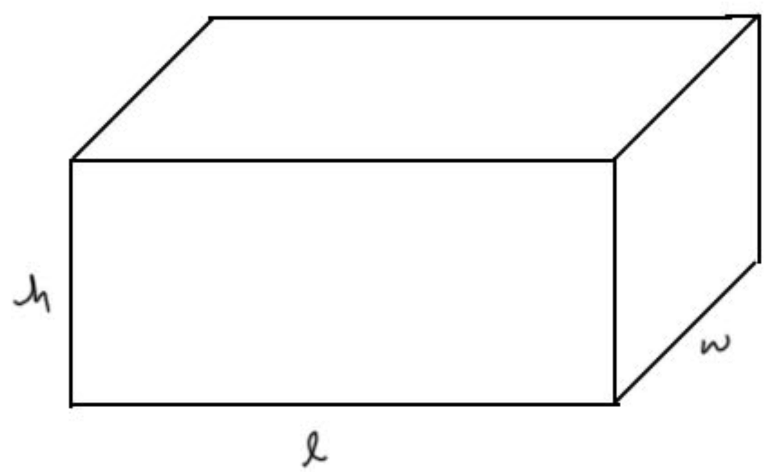
$(\pm\sqrt{5}, 0, 0)$ are the points on the surface closest to the origin



Ex. A jewelry box is to be made to hold $1000\ cm^3$ of jewels. The top is made of gold at \$200/cm^2, the front of silver at \$90 / cm^2, The sides bronze at \$50 / cm^2

and The back and bottom of iron at \$1 / cm^2.

Find The dimensions of The box of least cost.



$$l \cdot w \cdot h = 1000$$

$$\text{Cost} = 200\,lw + 90\,lh + 2(50)hw + lw + lh$$

$$= 201\,lw + 91\,lh + 100\,hw \qquad h = \frac{1000}{lw}$$

$$= 201\,lw + 91\,l\,\frac{1000}{lw} + 100\,\frac{1000}{lw}\,w$$

$$= 201\,lw + \frac{91000}{w} + \frac{100000}{l} = f(l,w)$$

want to minimize $f(l,w)$

$$f_l(l,w) = 201w - \frac{100000}{l^2} = 0 \qquad w = \frac{100000}{201l^2}$$

$$f_w(l,w) = 201l - \frac{91000}{w^2} = 0$$

$$201l - \frac{91000 \cdot 201^2 l^4}{(100000)^2} = 0$$

$$l - \frac{18291000\ l^4}{10^{10}} = 0$$

$$l - .0018291\, l^4 = 0$$

$$l(1 - .0018291\, l^3) = 0$$

$l = 0$ (no box!) $\qquad l = \sqrt[3]{\frac{1}{.0018291}} \approx 8.2$ cm

$w \approx 7.4$ cm

test to make sure it's a minimum:

$$f_{\ell\ell} = +\frac{200000}{\ell^3}$$

$$f_{ww} = +\frac{182000}{w^3}$$

$$f_{\ell w} = 201$$

$$D = \frac{200000 \cdot 182000}{(8.2)^3 \, (7.4)^3} - (201)^2$$

$$\left.\begin{array}{l} \approx 162916 - 40401 > 0 \\ \quad \text{with } f_{\ell\ell}(8.2, 7.4) > 0 \end{array}\right\} \text{min}$$

to answer question, need h

$$h = \frac{1000}{\ell w} \approx \frac{1000}{(8.2)(7.4)} \approx 16.5 \text{ cm.}$$

dimensions of the jewelry box of minimum cost That can hold $1000\,cm^3$ of jewels

is $\underset{l}{8.2\,cm} \times \underset{w}{7.4\,cm} \times \underset{h}{16.5\,cm}$.