Math 20300
Calculus III

Lesson 8
Quadric Surfaces

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Quadric Surfaces

A quadric surface is the graph of a second-degree equation in $x, y,$ and z.

(Three-dimensional version of conic sections in $\mathbb{R}^2$)

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$$

We'll look at 6 basic surfaces that each have one of the following forms: $Ax^2 + By^2 + Cz^2 + J = 0$

$$Ax^2 + By^2 + Iz = 0$$

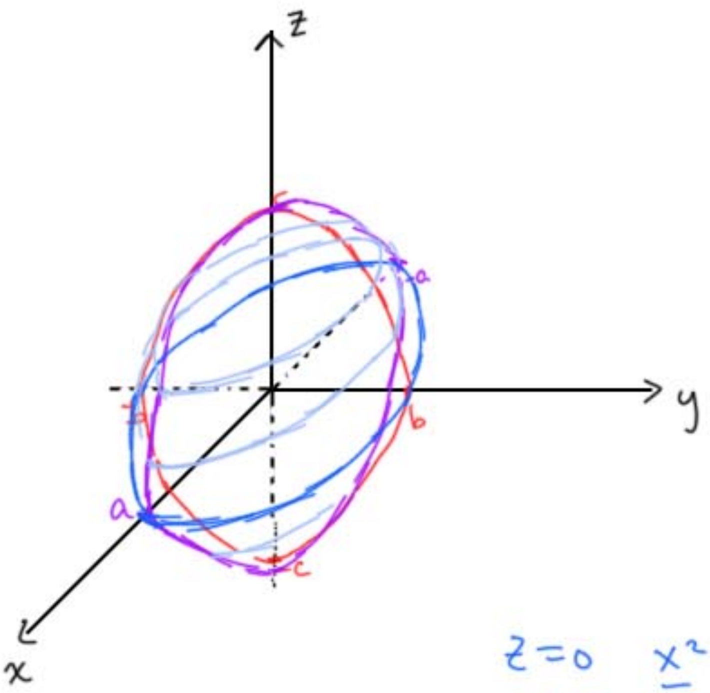
We can get the graphs of the more general equations by rotating + translating these.

Def. A <u>trace</u> of a surface is the curve obtained by intersecting the surface with a plane parallel to the coordinate planes.

$x = \text{constant}$,
$y = \text{constant}$
or $z = \text{constant}$

Ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

If $a=b=c$, this is a sphere.



All traces are ellipses

$x=0$ $\dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$

ellipse in yz plane

$y=0$ $\dfrac{x^2}{a^2} + \dfrac{z^2}{c^2} = 1$

ellipse in xz plane

$z=0$ $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ ellipse in xy plane

Elliptic Paraboloid

$$z = ax^2 + by^2 + c$$

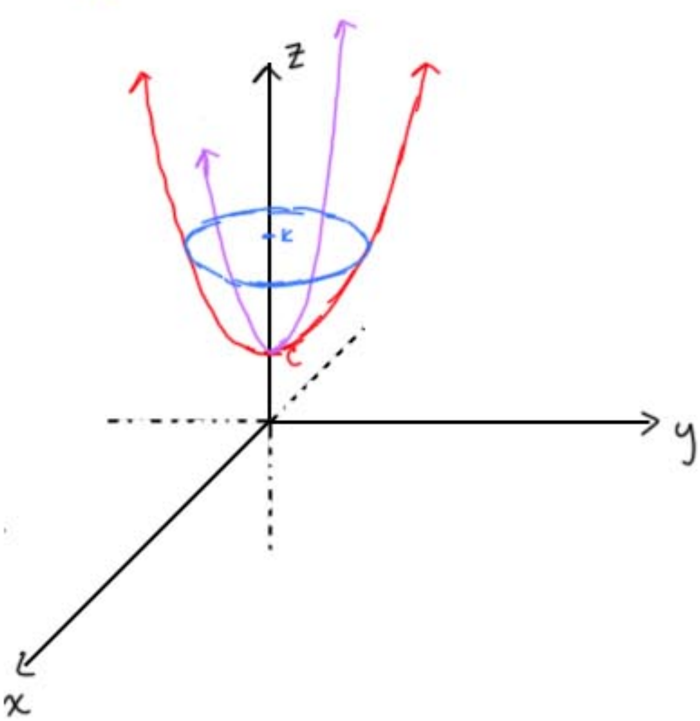
$(a, b > 0)$

$x=0$
$z = by^2 + c$

$y=0$
$z = ax^2 + c$

$z = k > c$
ellipse

vertical traces are parabolas
horizontal traces are ellipses



Hyperbolic Paraboloid

$z = ax^2 - by^2 + c$

$(a, b > 0)$

vertical traces are parabolas

horizontal traces are hyperbolas for $z = k \neq c$

lines $y = \pm\sqrt{\frac{a}{b}}\, x$ for $z = c$

$x = 0$

$z = -by^2 + c$ ← choosing $c = 0$ here

$y = 0$

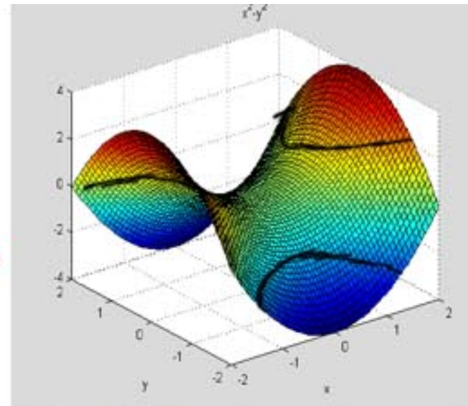
$z = ax^2 + c$

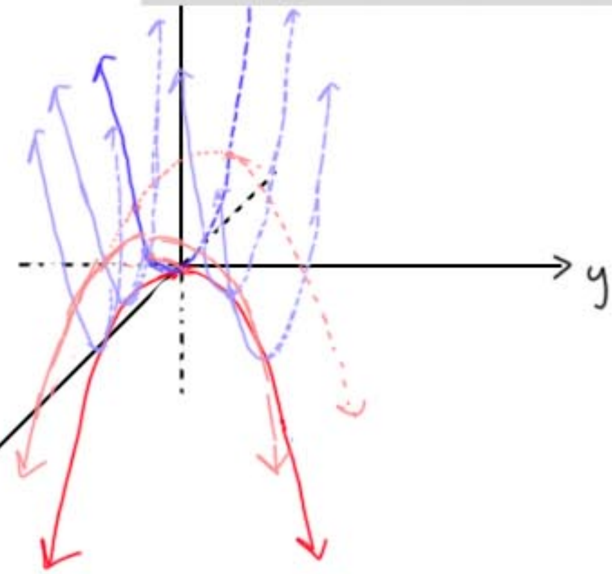
$y = k$

$z = ax^2 - bk^2 + c$

$x = k$

$z = ak^2 - by^2 + c$





Cone

$z^2 = ax^2 + by^2$

$(a, b > 0)$

vertical traces $x = k$ and $y = k$ are hyperbolas for $k \neq 0$,

pairs of lines for $k = 0$

horizontal traces are ellipses

$x = 0$

$z^2 = by^2$

$z = \pm\sqrt{b}\, y$

lines

$y = 0$

$z^2 = ax^2$

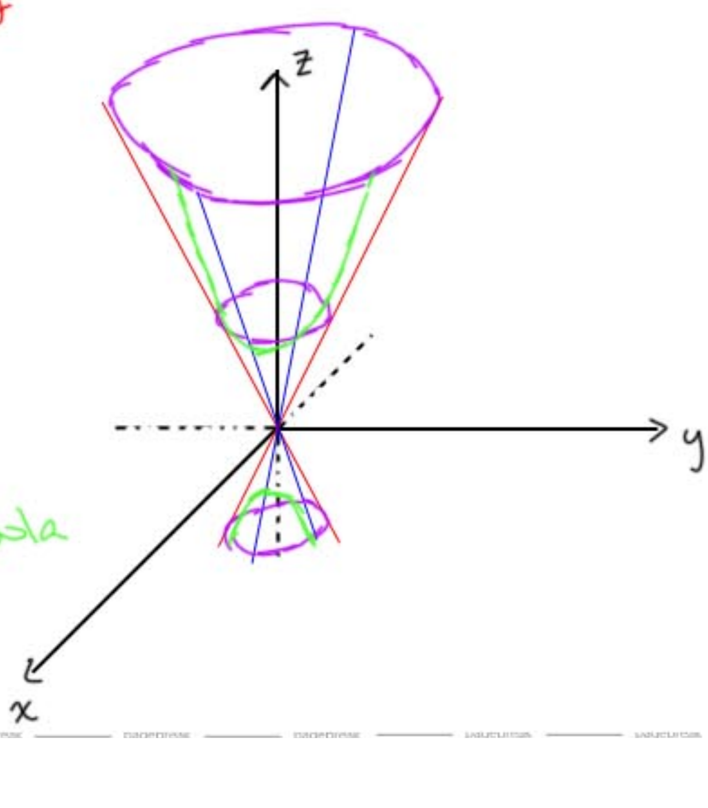
$z = \pm\sqrt{a}\, x$

lines

$z = k \neq 0$

$x = k \neq 0$

hyperbola



Hyperboloid of one sheet

$ax^2+by^2-cz^2=1$

$(a,b,c>0)$

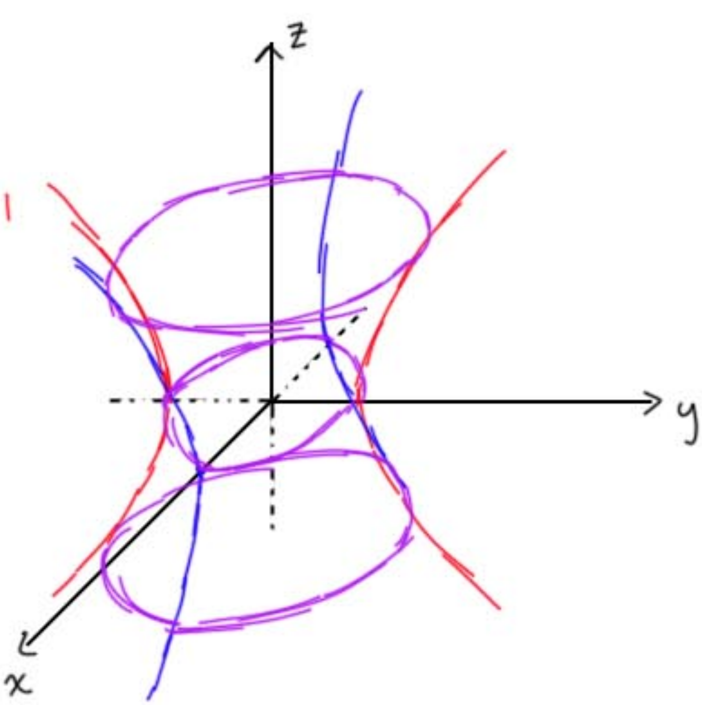
$x=0$
$by^2-cz^2=1$
hyperbola

vertical traces are hyperbolas

$y=0$
$ax^2-cz^2=1$
hyperbola

horizontal traces are ellipses

$z=0$
$ax^2+by^2=1$



Hyperboloid of two sheets

$ax^2-by^2-cz^2=1$

$(a,b,c>0)$

positive coeff.

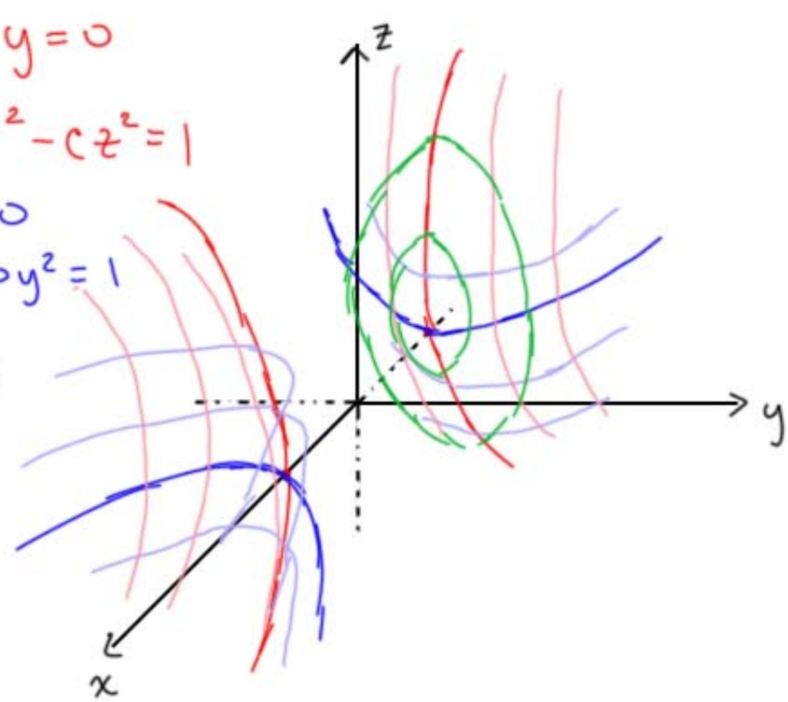
$y=0$
$ax^2-cz^2=1$

$z=0$
$ax^2-by^2=1$

traces $x=k$ $k>\frac{1}{\sqrt{a}}$ or $k<-\frac{1}{\sqrt{a}}$ ellipses

traces $y=k$, $z=k$ hyperbolas

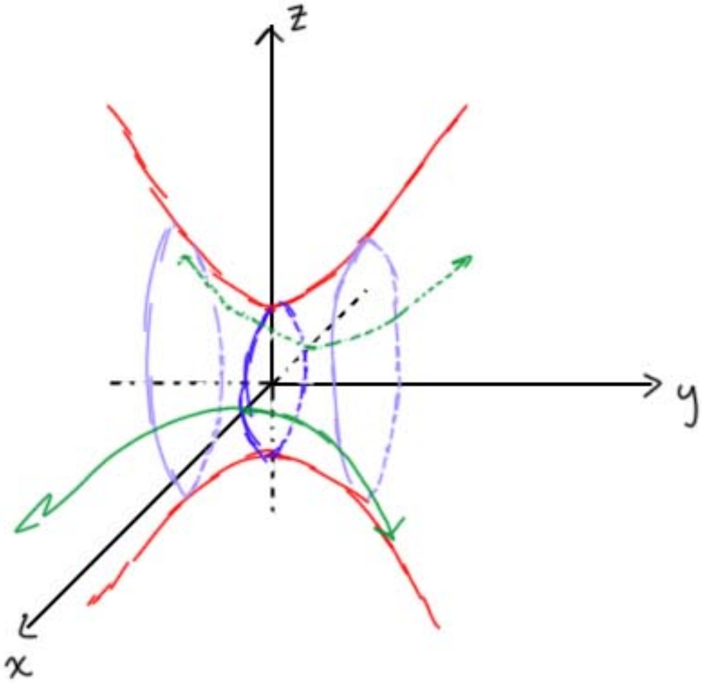
$x=k$ ellipses



Ex. Sketch the appropriate traces Then

Sketch and identify the surface.

$$2x^2 - y^2 + 3z^2 = 1$$



$x = 0$

$-y^2 + 3z^2 = 1$

hyperbola in yz plane

$y = 0$

$2x^2 + 3z^2 = 1$

ellipse

$y = k$

$2x^2 + 3z^2 = 1 + k^2$

ellipse

$z = 0$

$2x^2 - y^2 = 1$

hyperbola

The surface is a hyperboloid of one sheet.

Ex. Sketch the appropriate traces then sketch and identify the surface.

$z = y^2 - x^2 + 2$

$x = 0$

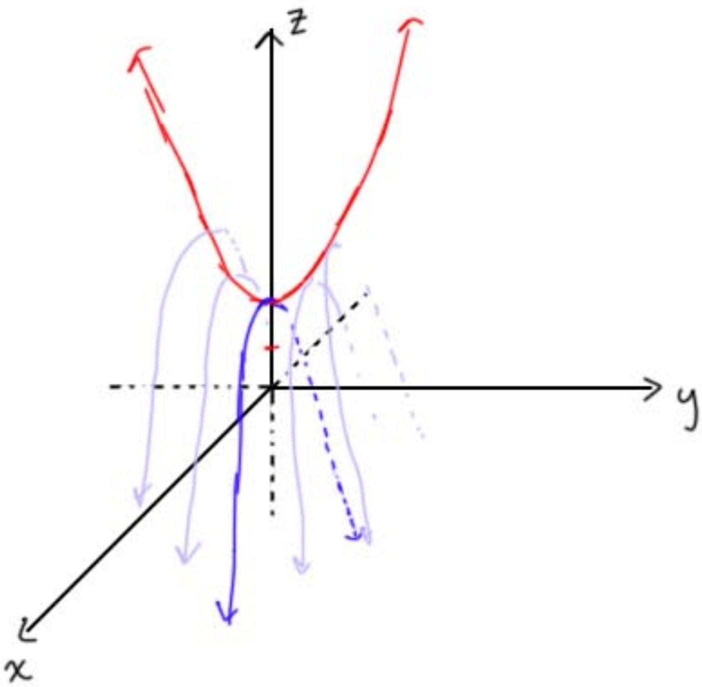
$z = y^2 + 2$ parabola

$y = 0$

$z = -x^2 + 2$

parabola

$y = k$



saddle surface, hyperbolic paraboloid.