Math 20200
Calculus II

Lesson 24
Polar Coordinates

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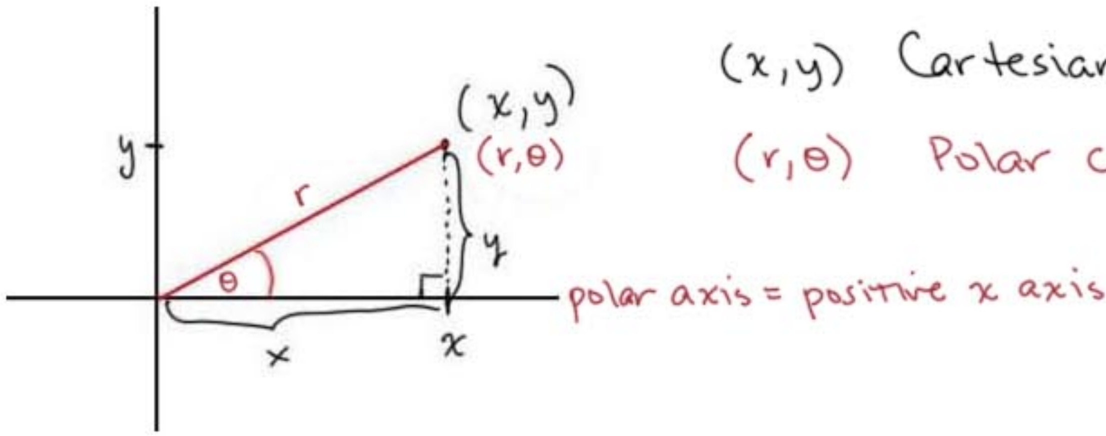
Table of Contents:

Polar Coordinates



(x, y) Cartesian coordinates

(r, θ) Polar coordinates

r = distance from the point to the origin (pole)

θ = angle made by the ray with the polar axis (positive x axis)

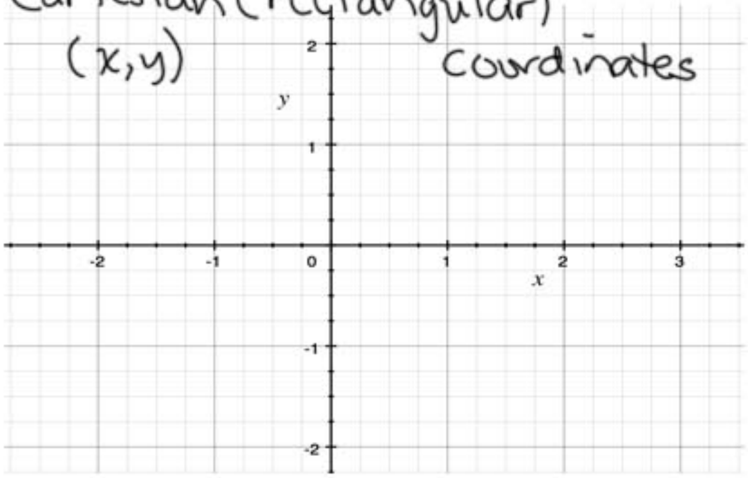
What is the relationship between (x,y) and (r,θ)?

from the picture above, we have $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$

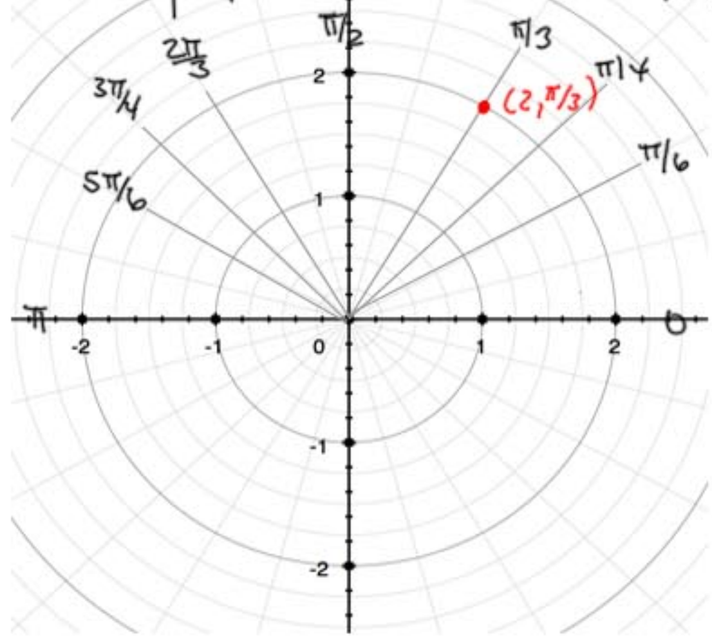
$\therefore$ $\boxed{y = r\sin\theta}$, $\boxed{x = r\cos\theta}$ $\Rightarrow$ $\boxed{\frac{y}{x} = \tan\theta}$

also, $\boxed{x^2 + y^2 = r^2}$.

Cartesian (rectangular) (x,y) coordinates



polar coordinates (r,θ)



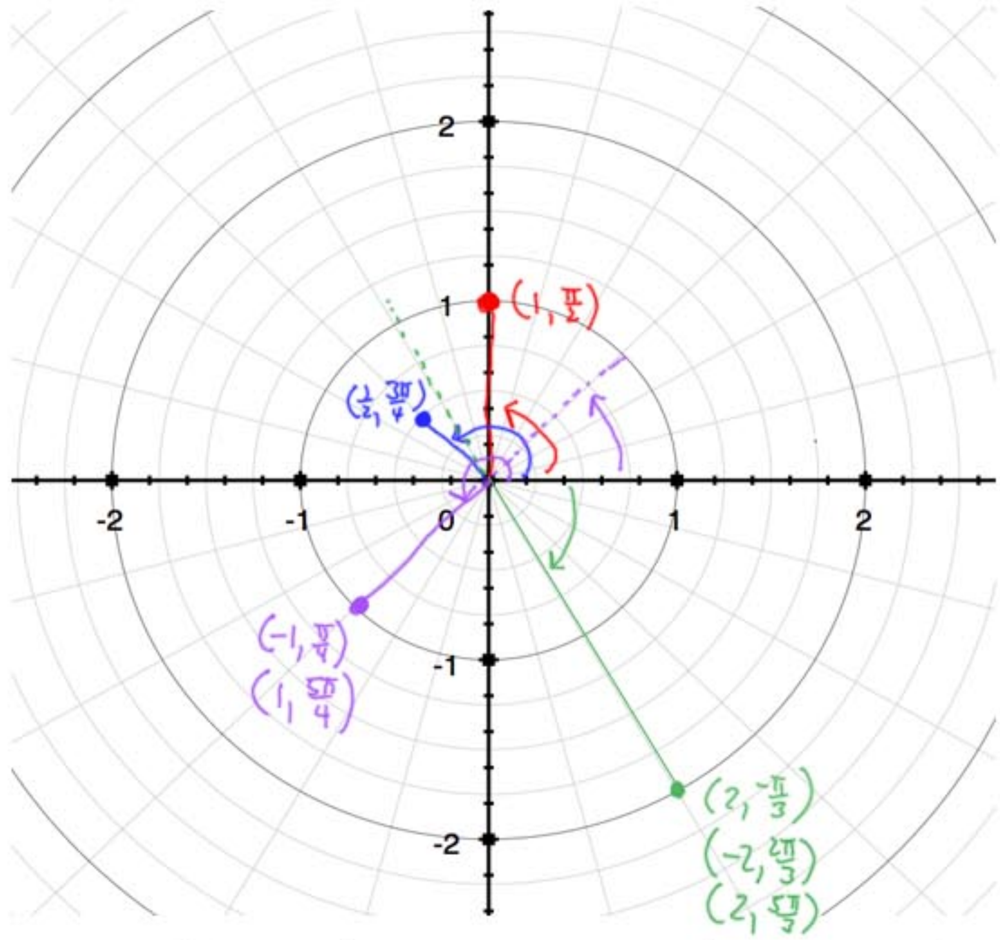
Plotting points in polar coordinates:

a. $(r,\theta)=(1,\frac{\pi}{2})$

b. $(r,\theta)=(\frac{1}{2},\frac{3\pi}{4})$

c. $(r,\theta)=(2,-\frac{\pi}{3})$

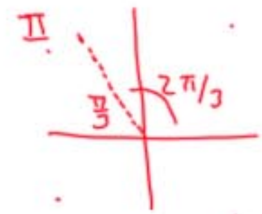
d. $(r,\theta)=(-1,\frac{\pi}{4})$



Note: polar representation of points is <u>not</u> unique!

Ex. Convert to Cartesian coordinates:

$$(r, \theta) = \left(2, \tfrac{2\pi}{3}\right).$$



$$x = r\cos\theta = 2\underbrace{\cos\tfrac{2\pi}{3}} = 2\left(-\tfrac{1}{2}\right) = -1$$

$$= 2\left(-\cos\tfrac{\pi}{3}\right)$$

QII ref angle

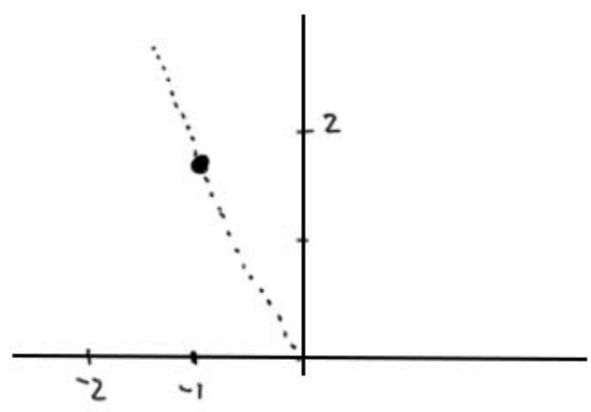
$$y = r\sin\theta = 2\sin\tfrac{2\pi}{3} = 2\left(\tfrac{\sqrt{3}}{2}\right) = \sqrt{3}.$$

$$2\left(+\sin\tfrac{\pi}{3}\right)$$

QII ref ∡

$$\therefore\ (x, y) = (-1, \sqrt{3}).$$

helps to plot both:



Ex. Convert to Cartesian coordinates:

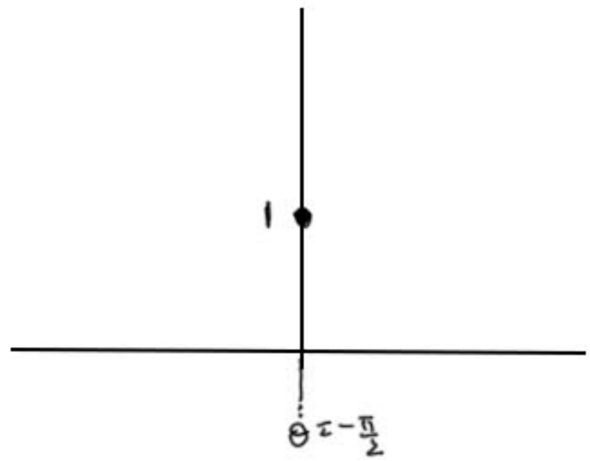
$$(r, \theta) = \left(-1, -\tfrac{\pi}{2}\right)$$

$$x = r\cos\theta = -1\cos\left(-\tfrac{\pi}{2}\right) = -1(0) = 0$$

$$y = r\sin\theta = -1\sin\left(-\tfrac{\pi}{2}\right) = -1(-1) = 1$$

$\therefore (x,y) = (0,1)$

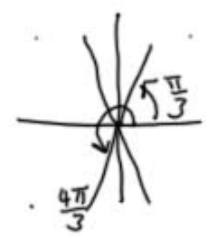
plot :



Ex. For $(x,y) = (-1, -\sqrt{3})$,

a) find (r,θ) with $r>0$ and $\theta \in [0, 2\pi)$
b) find (r,θ) with $r<0$ and $\theta \in [0, 2\pi)$

to find θ, $\tan\theta = \frac{y}{x} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$



$\therefore \theta = \frac{\pi}{3}, \frac{4\pi}{3}$ ← $\tan\theta > 0$ in Q I, III with $\frac{\pi}{3}$ reference angle.

↖ memorized from common 1st quadrant angles

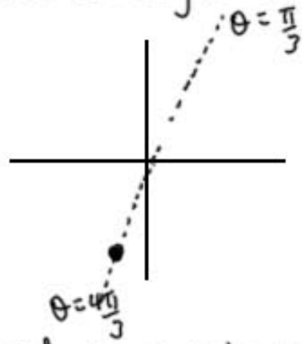
then $r^2 = x^2 + y^2 = (-1)^2 + (-\sqrt{3})^2 = 1 + 3 = 4$

$r^2 = 4 \Rightarrow r = \pm 2$

How do we know which θ goes with which r?

notice $(-1, -\sqrt{3})$ is in Q III,

so for $\theta = \frac{\pi}{3}$, we need a negative r b) $(r,\theta) = (-2, \frac{\pi}{3})$



and for $\theta = \frac{4\pi}{3}$, we need a positive r a) $(r,\theta) = (2, \frac{4\pi}{3})$