# Math 20100
# Calculus I

# Lesson 23
# Sigma Notation

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# Sigma Notation

Sigma notation is used to conveniently denote a summation:

$i=1 \quad i=2 \quad i=3 \quad i=4 \quad i=5 \quad i=6 \quad i=7$

$$1+2+3+4+5+6+7 = \sum_{i=1}^{7} i$$

sigma means sum

counting index

integers

$$1+2+3+4+5+6+7 = \sum_{i=0}^{6} i+1$$

$i=0 \quad i=1 \quad i=2 \quad i=3 \quad i=4 \quad i=5 \quad i=6$

same sum, different indexing

$i=1 \quad i=2 \quad i=3 \quad i=4 \quad i=5$

$$2+4+6+8+10 = \sum_{i=1}^{5} 2i$$

$$2(1+2+3+4+5) = 2\sum_{i=1}^{5} i$$

Ex. $$\sum_{i=0}^{4}(2i-1) = \overset{i=0}{(2(0)-1)} + \overset{i=1}{(2(1)-1)} + \overset{i=2}{(2(2)-1)}$$

$$+ \overset{i=3}{(2(3)-1)} + \overset{i=4}{(2(4)-1)}$$

$$= -1+1+3+5+7 = 15.$$

Ex. $$\sum_{i=3}^{7}(i^2+i) = \overset{i=3}{(3^2+3)} + \overset{i=4}{(4^2+4)} + \overset{i=5}{(5^2+5)} +$$

$$+ \overset{i=6}{(6^2+6)} + \overset{i=7}{(7^2+7)}$$

$$= 12+20+30+42+56 = 160.$$

Ex. Write in summation notation

$$\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \sqrt{7}$$

$$= \sum_{i=3}^{7} \sqrt{i} \quad \text{OR} \quad = \sum_{i=0}^{4} \sqrt{i+3}$$

infinitely many ways to write the summation,

Summation notation is <u>not</u> unique

Ex. $1 + 4 + 9 + 16 + 25 + 36 = \displaystyle\sum_{i=1}^{6} i^2$

$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$

Ex. $x + x^2 + x^3 + \cdots + x^n = \displaystyle\sum_{i=1}^{n} x^i$

$i=1 \quad i=2 \quad i=3 \quad \cdots \quad i=n$

Some useful formulas

$$\sum_{i=1}^{n} c = cn \qquad \text{Ex. } \sum_{i=1}^{4} 5 = 5+5+5+5$$
$$= 5(4) = 20$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \text{Ex. } \sum_{i=1}^{25} i = \frac{25(26)}{2}$$
$$= 25(13) = 325$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\text{Ex. } \sum_{i=1}^{10} i^2 = \frac{\cancel{10}^{5}(11)(\cancel{21}^{7})}{\cancel{6}\,\cancel{3}} = 385$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

$$\text{Ex. } \sum_{i=1}^{20} i^3 = \frac{20^2(21)^2}{4} = 100 \cdot 441 = 44{,}100.$$

Ex. $$\sum_{i=4}^{20} i^3 = \sum_{i=1}^{20} i^3 - \sum_{i=1}^{3} i^3$$

be careful: to use the formulas, the summation needs to start at i=1

$$= 44{,}100 - \frac{3^2(4)^2}{4} =$$

$$= 44{,}100 - 36$$

$$= 44{,}064$$

Ex. $$\sum_{i=1}^{n} (3+2i)^2 = \sum_{i=1}^{n} (9 + 12i + 4i^2)$$

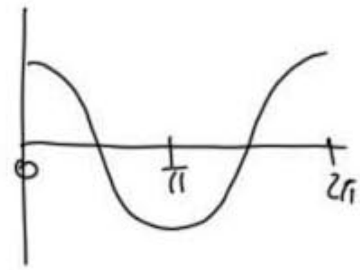
$$= \sum_{i=1}^{n} 9 + \sum_{i=1}^{n} 12i + \sum_{i=1}^{n} 4i^2$$

$$= \sum_{i=1}^{n} 9 + 12 \sum_{i=1}^{n} i + 4 \sum_{i=1}^{n} i^2$$

$$= 9n + 12 \cdot \frac{n(n+1)}{2} + 4 \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= 9n + 6n(n+1) + \frac{2}{3}n(n+1)(2n+1).$$

Ex. $$\sum_{k=0}^{8} \cos(k\pi) =$$




$$= \underbrace{\cos(0\pi)}_{1} + \underbrace{\cos(1\pi)}_{-1} + \underbrace{\cos(2\pi)}_{1} + \underbrace{\cos(3\pi)}_{-1} + \underbrace{\cos(4\pi)}_{1} +$$

$$+ \underbrace{\cos(5\pi)}_{-1} + \underbrace{\cos(6\pi)}_{1} + \underbrace{\cos(7\pi)}_{-1} + \underbrace{\cos(8\pi)}_{1} = 1.$$

Ex. $$\sum_{k=3}^{8} \sin\left(\frac{k\pi}{2}\right) =$$


Work on this problem
on your own

$$= \sin\left(\frac{3\pi}{2}\right) + \sin\left(\frac{4\pi}{2}\right) + \sin\left(\frac{5\pi}{2}\right) + \sin\left(\frac{6\pi}{2}\right) + \sin\left(\frac{7\pi}{2}\right) + \sin\left(\frac{8\pi}{2}\right)$$

$$= -1 + 0 + 1 + 0 + -1 + 0$$

$$= -1.$$