

1. Use Newton's method to approximate a of the nonlinear system $0 = f_1(x_1, x_2) = x_1^2 - 2x_1 - x_2 + 1$ and $0 = f_2(x_1, x_2) = x_1^2 + x_2^2 - 1$.
 - (a) $x_1 = (\frac{3}{2}, 0)$ and $x_2 = \frac{1}{12}(3, -2)$ when $\vec{x}_0 = (1, 1)^T$.
 - (b) Write code to find $x_4 = [1.00000003, -1.54953310e - 08]$ when $\vec{x}_0 = (1, 1)^T$. This FPI converges to $(1, 0)$.
 - (c) $x_1 = \frac{1}{3}(-1, 4)$ when $\vec{x}_0 = (-1, 2)^T$. Use Numpy to approximate $x_5 = [-2.32830616e-10, 1.00000000]$. This FPI converges to $(0, 1)$.
 - (d) What goes wrong when the intial guess is $\vec{x}_0 = (0, 0)^T$? The jacobian J is singular at $(0, 0)$ and it is impossible to solve $Jp_k = F(x_k)$.
2. Use Newton's method to approximate a of the nonlinear system $0 = f_1(x_1, x_2) = x_1^2 + x_2^2 - 1$ and $0 = f_2(x_1, x_2) = x_2 - x_1^3$.
 - (a) Find $x_1 = (1, 1)$ by hand when $\vec{x}_0 = (1, 2)^T$.
 - (b) Write code to find $x_5 = [0.82603136, 0.56362416]$ when $\vec{x}_0 = (1, 2)^T$. Does this FPI converge? Yes it seems to have stabilized 8 digits after decimal place at $x_5 = [0.82603136, 0.56362416]$.
 - (c) Find $x_1 = (\frac{5}{4}, -1)$ by hand when $\vec{x}_0 = (2, 0)^T$. Use Numpy to approximate $x_4 = [-1.07763436, -0.98422872]$. This FPI seems to converge to $x_8 = [-0.82603136, -0.56362416]$ to 8 decimal places.