1.

2. Let $A = \begin{bmatrix} -1 & 2 & 2 \\ -1 & -4 & -2 \\ -3 & 9 & 7 \end{bmatrix}$.

   (a) Find the eigenvalues and eigenvectors of A by hand. $\lambda_1 = 3, v_1 = (1, -1, 3), \lambda_2 = -2, v_2 = (0, 1, -1)$, and $\lambda_3 = 1, v_3 = (-1, 1, -2)$.

   (b) Use the power method by hand with initial guess $v_0 = (1, 0, 0)$ to find $v_1 = \frac{1}{\sqrt{11}}(-1, -1, 3)$ and $\lambda_1 = 7$.

   (c) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 0, 0)$ to find $v_1 = \frac{(6,-13,21)}{\sqrt{6^2+13^2+21^2}}$.

   (d) Use the inverse power method by hand with $\alpha = -1$ and with initial guess $v_0 = (1, 0, 0)$ to find $v_1 = \frac{(3,-7,9)}{\sqrt{3^2+7^2+9^2}}$.

3. Let $A = \begin{bmatrix} -1 & 2 & 2 \\ -1 & -4 & -2 \\ -3 & 9 & 7 \end{bmatrix}$.

   (a) Use the power method with initial guess $v_0 = (1, 0, 0)$ to find $v_{11} = [-0.30278232, 0.29928181, -0.90484986]$ and $\lambda_{11} = 3.02109187541328$.

   (b) Use the inverse power method with $\alpha = 0$ and with initial guess $v_0 = (1, 0, 0)$ to find $v_9 = [0.40805248, -0.40845098, 0.81649309]$ and $\lambda_9 = 0.998467873996931$.

   (c) Use the inverse power method with $\alpha = -1.7$ and with initial guess $v_0 = (1, 0, 0)$ to find $v_8 = [0.00557856, 0.70987412, -0.70430648]$ and $\lambda_8 = -2.035276365005557$.

4. Let $A = \begin{bmatrix} 7 & 4 \\ 3 & 6 \end{bmatrix}$.

   (a) Find the eigenvalues and eigenvectors of A by hand. $v_1 = (4, 3), \lambda_1 = 10$ and $v_2 = (-1, 1), \lambda_2 = 3$.

   (b) Use the power method by hand with initial guess $v_0 = (1, 1)$ to find $v_1 \frac{(11,9)}{\sqrt{202}}$ and $\lambda_1 = \frac{113*11+87*9}{202}$.

   (c) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 1)$ to find $v_1 = \frac{(1,2)}{\sqrt{5}}$ and $\lambda_1 = 9$.

5. Let $A = \begin{bmatrix} 7 & 4 \\ 3 & 6 \end{bmatrix}$.

   (a) Use the power method by hand with initial guess $v_0 = (1, 1)$ to find $v_{18} = [0.8, 0.6]$ and $\lambda_{18} = 10.000000000054238$.

   (b) Use the inverse power method by hand with $\alpha = 0$ and with initial guess $v_0 = (1, 1)$ to find $v_{13} = [-0.70710599, 0.70710757]$ and $\lambda_{13} = 2.9999988839824407$.

6. Find the singular values of each of the following matrices.

   (a) $\sigma_1 = 2, \sigma_2 = 1$.

   (b) $\sigma_1 = 5, \sigma_2 = 0$.

   (c) $\sigma_1 = \sqrt{3}, \sigma_2 = 1$.

7. Find the SVD of each of the following matrices.

   (a) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

   (b) $A = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 0 \end{bmatrix} \frac{1}{\sqrt{5}} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

(c) $V = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \end{bmatrix}, U = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $\Sigma = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$.