1. (a) $x_1 = (1,1,0$ and $x_2 = (0,1,2)$ are independent vectors that are perpendicular to $a = (2,-2,1)$.
   (b) The angle between $a = (2,-2,1)$, and $b = (1,2,2)$ is $\pm\frac{\pi}{2}$.
   (c) $(0,0,0)$ is the projection of $b = (1,2,2)$ onto span $a = (2,-2,1)$?
   (d) $\frac{8}{9}a$ is the projection of $b = (1,-2,2)$ onto span $a = (2,-2,1)$? $\hat{r} = b - Pb = \frac{1}{9}(-7,-2,10)$.
   (e) $(1,1,0)$ is the projection of $b = (1,1,1)$ onto the plane spanned by $(1,0,0)$ and $(1,1,0)$. $\hat{r} = b - Pb = (0,0,1)$.

2. Geometry of matrix multiplication as a linear transformation.
   (a) Given $b = (1,2,2)$ and $a = (1,1,1)$. Then $A\hat{x} = Pb = \frac{1}{3}(5,5,5), \hat{r} = b - Pb = \frac{1}{3}(-2,1,1)$.
   (b) Find the best least squares solution $\hat{x} = 2$ to $3x = 10, 4x = 5$. How is the residual minimized? The $2-$norm of the residual is minimized. The residual $\hat{r} = b - A\hat{x} = (4,-3)$ is perpendicular to the column of A. Check with dot product.
   (c) Solve $Ax = b$ by least squares when $A = \begin{bmatrix}1 & 0\\0 & 1\\1 & 1\end{bmatrix}, b = \begin{bmatrix}1\\1\\0\end{bmatrix}$. $\hat{x} = \frac{1}{3}(1,1)$ and $\hat{r} = b - A\hat{x} = \frac{1}{3}(2,2,-2)$ is perpendicular to the columns of A.

3. Geometry of matrix multiplication as a linear transformation.
   (a) Project the vector $b = (1,1,8)$ onto the columns of $A = \begin{bmatrix}1 & 1\\0 & 1\\1 & -1\end{bmatrix}$. $Pb = \frac{1}{2}(3,-6,15)$ and $\hat{r} = b - Pb = \frac{1}{2}(-1,8,1)$ is perpendicular to the columns of A.
   (b) Find the projection matrix $P = A(A^TA)^{-1}A^T = \frac{1}{2}\begin{bmatrix}2 & 1 & 0\\1 & 1 & -1\\0 & -1 & 2\end{bmatrix}$
   (c) Redo the first question on this page using the projection matrix P. Compute Pb by matrix multiplication.

4. (a) Find the least squares solution to Ax=b when $A = \begin{bmatrix}6 & 9\\3 & 8\\2 & 10\end{bmatrix}, b = \begin{bmatrix}0\\49\\0\end{bmatrix}$. Then determine the $2-$norm of the residual. ANSWER $\hat{x} = (-1,2)$ and $\|\hat{r}\|_2 = 42$.
   (b) Fit a linear function of the form $f(t) = b + mt$ to the data points $(0,3), (1,3)$, and $(1,6)$. ANSWER: $3 + 1.5t$.

5. Use Householder reflectors to find the QR factorization of the following matrices.
   (a) ¡2-¿ $A = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix} = \begin{bmatrix}0.6 & 0.8\\0.8 & -0.6\end{bmatrix}\begin{bmatrix}5 & 3\\ & -1\end{bmatrix} = QR$
   (b) ¡3-¿ $A = \begin{bmatrix}1 & -4\\2 & 3\\2 & 2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 2\\2 & 1\\2 & -2\end{bmatrix}\begin{bmatrix}3 & 2\\ & -3\end{bmatrix}$.