1. Determine if each function has a unique fixed point in the given interval.
   (a) $g(x) = 1 - \frac{x^2}{4}$ on $[0, 1]$. It has a fixed point by the applying the IVP to $f(x) = 1 - \frac{x^2}{4} - x$ on $[0, 1]$. Since $|g'(x)| \leq \frac{1}{2}$ on $[0, 1]$ there is only one fixed point.
   (b) $g(x) = 2^{-x}$ on $[0, 1]$. Use a similar argument as (a) to show there is one and only one fixed point in $[0, 1]$.
   (c) $g(x) = \frac{1}{x}$ on $[0.5, 5.2]$. You can see there is one fixed point by graphing $g(x)$ and $y = x$. FPI may not converge because $|g'(1)| = 1$.

2. Use FPI when $g(x) = -4 + 4x - \frac{x^2}{2}$. Find the actual errors and the actual relative errors for each iterate.
   (a) $x_1 \approx 1.7949, x_2 \approx 1.5689, x_3 \approx 1.0450$ when $x_0 = 1.9$.
   (b) $x_1 \approx 3.9799, x_2 \approx 3.9997, x_3 \approx 3.9999$ when $x_0 = 3.8$.
   (c) $|g'(2)| = 2 > 1$ so FPI will diverge when x_0 is near the fixed point $x^* = 2$. $|g'(4)| = 0$ so FPI will converge fast if x_0 is near the fixed point $x^* = 4$.

3. Use FPI when $g(x) = 0.5x + 1.5$. Find the actual errors and the actual relative errors for each iterate.
   (a) Find $x_1 = 3.5, x_2 = 3.25, x_3 = 3.125, x_4 = 3.0625, x_5$ when $x_0 = 4$.
   (b) Fixed point $x^* = 3$.
   (c) Can FPI be used to find a fixed point of $g(x) = x^2 + x - 4$. Explain. FPI will most likely diverge from both fixed points since $|g'(2)| = 5 > 1$ and $|g'(-2)| = 3 > 1$.

4. Make cobweb (staircase) diagrams for each of the following. Make a rough sketch. This is not a good way to approximate the answer, just a rough sketch to see what is going on.

5. $f(x) = x \sin x$ has a root x^* between $a = 2$ and $b = 4$ because it is continuous and
   (a) Use the Bisection Method for f, a and b to find $x_8 = 3.14453125$.
   (b) Use the Bisection error formula to find an error bound for your approximate root x_8. $e_8 \leq \frac{1}{2^8}$.
   (c) How many steps of Bisection are necessary to get 11 digits of accuracy after the decimal place? 44 steps are sufficient since it takes at most 4 steps of Bisection to stabilize one digit.

6. Let $f(x) = e^x - x - 2$.
   (a) $a = 0$ and $b = 2$ gives $f(a) \cdot f(b) < 0$.
   (b) Use the Bisection Method for f, a and b to find $x_8 = 1.14453125$.
   (c) Use the Bisection error formula to find an error bound for your approximate root x_{18}. $e_{18} \leq \frac{1}{2^{18}}$.
   (d) How many steps of Bisection are necessary to get 9 digits of accuracy after the decimal place? 36 iterations will safely give 9 digits of accuracy.

7. (a) $x^* = \sqrt{5}$ can be approximated by using Newton's method on $0 = f(x) = x^2 - 5$ giving $x = g(x) = x - \frac{x^2-5}{2x}$. In this case $x_3 = 2.236067977915804$ when $x_0 = 2$.
   (b) Use the Secant Method to find the approximate roots $x_2 = -2.1065989847715736, x_3 = -2.0226414123070677$, and $x_4 = -2.0015110973304853$ to $f(x) = x^3 - 3x + 2$ when $x_0 = -2.6$ and $x_1 = -2.4$.

8. (a) Use Newton's method to find the approximation $x_{11} = 2$ to a root of $f(x) = x^3 - 3x - 2$ when $x_0 = 2$. Give your answer with 10 decimal places of accuracy.
   (b) Repeat the above problem with different initial approximations $x_0 = -3$ then $x_{11} = -1.0015415688887797$. Newton's method converges slower to the root $x^* = -1$ than to the root $x^* = 2$ because $f'(-1) = 0$ and this is an obvious problem in Newton's Method.