HW QUIZ 1

1. (8 points) Suppose the spread of an illness similar to measels is modelled by the following rate equations:

$$S' = -.0002SI$$
$$I' = .0002SI - .08I$$
$$R' = .08I$$

   (a) Roughly how long does someone who catches the illness remain infected?
   (b) How large does the susceptible population have to be in order for the illness to take hold–that is for the number of cases to increase? Explain your reasoning. during the next 24 hours?
   (c) Approximate $S(1), I(1), R(1)$ using $\Delta t = 1$ when intial values are $S = 10000, I = 500, R = 800$.

2. (2 points) The following program prints 4 lines of output. What are they?

```
a = 0
b = 0
for k in range(3):
    a += 1
    b += a
    print(a,b)
print(2*a+b)
```

HW QUIZ 2 (with computer)

1. (6 points) How would you modify SIRVALUE to calculate estimates for S, I, and R when $t = 120$ days and $\Delta t = 0.25$. Do it; what do you get? Give your approximate answers rounded to two decimal places.

2. (3 points) The following program prints 4 lines of output. What are they?

```
a = 10
b = 0
for k in range(3):
    a += 1
    b -= a
    print(a,b)
print(2*a+b)
```

3. (2 points) Modify your SIRVALUE code to approximate $y(2.5)$ when $y(t)$ solves the IVP

$$y' = .2y(5 - y), y(0) = 1$$

. How small must Δt be in order that your approximation of $y(2.5)$ have two decimal places of accuracy. Explain with a table of approximations of $y(2.5)$ for different δt values.

HW QUIZ 3

1. (5 points) Construct the Euler approximation to $y(1)$ with $\Delta t = \frac{1}{2}$ and when the function $y(t)$ that solves the IVP:
$$y' = \frac{4}{1+t^2}, y(0) = 0.$$

2. (5 points) Find two successive approximations to $\sqrt{2}$ using the BABYLON program, an algorithm realized by a FOR loop with a single line that reads

```
x=(x + 2/x) / 2
```

(You should start with $x = 1$).

HW QUIZ 4

1. (5 points) Find the largest interval in which x must lie in order to approximate 900 with relative error at most 10^{-3}.

2. (5 points) Find the maximum $\max_{0 \leq x \leq 4} |f(x)|$ for
$f(x) = x^2\sqrt{(4-x)}$.

HW QUIZ 5 (with computer)

1. (10 points) Use the Bisection method to find a solution accurate to within 10^{-2} for $x^4 - 2x^3 - 4x^2 + 4x + 4 = 0$ on the interval $[0, 2]$. Explain why your solution attains this accuracy.

HW QUIZ 6

1. Let $g(x) = 2^{-x}$.
   (a) (3 points) Determine an interval $[a, b]$ on which the fixed point iteration $x = g(x)$ will converge.
   (b) (3 points) Draw a cobweb (staircase) diagram to approximate the fixed point $x = g(x)$.
   (c) (3 points) Find x_2 by applying FPI to $x = g(x)$ with initial guess $x_0 = 0$.
   (d) (1 point) Estimate the number of iterations necessary in part (c) to approximate the fixed point accurate to within 10^{-2}. (HINT: $\ln 2 \approx 0.69$). Explain.

HW QUIZ 7

1. (10 points) Let $f(x) = -x^3$ and $x_0 = -1$. Use Newton's method to find x_1, and x_2.

HW QUIZ 8

1. (10 points) Solve the system $\begin{bmatrix} 3 & 7 \\ 6 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -11 \end{bmatrix}$ by finding the LU factorization and then carrying out forward and backwards substitution.

HW QUIZ 9

1. (4 points) When $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, use the normal equations to find the least squares solution of $Ax = b$ when
2. (3 points) Write one line of code to replace the third row of a given matrix A by itself minus 4 times the first row.
3. (3 points) Find P, L, and U to decompose $A = \begin{bmatrix} 2 & 1 & 5 \\ 4 & 4 & -4 \\ 1 & 3 & 1 \end{bmatrix}$ as $PA = LU$ using partial pivoting.

HW QUIZ 10

1. (5 points) (a) Find the QR factorization of the matrix $A = \begin{bmatrix} 6 & 2 \\ 3 & -6 \\ 2 & 3 \end{bmatrix}$.

   (b) Use the factorization from part (a) to find the least squares solution to $Ax = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

2. (5 points) Compute $\vec{x}_1$ when $\vec{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ using the Jacobi method to approximate a solution to $Ax = b$ when $A = \begin{bmatrix} 3 & -1 \\ -1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$. Compute the residual and the relative backward error for the approximation $\vec{x}_1$.

HW QUIZ 11

1. (5 points) Given the 1000×1000 matrix A, approximate how many flops are required to solve the 500 problems $Ax = b_1, Ax = b_2, \ldots, Ax = b_{500}$.

2. (5 points) Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 3 & 1 & 0 \\ 4 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

   (a) Solve $Ax = b$ when $b = \begin{bmatrix} 1 \\ 1 \\ 4 \\ 5 \end{bmatrix}$.

   (b) Solve $Ax = b$ when $b = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$.

HW QUIZ 12

1. (10 points) Let $A = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$. Solve $Ax = b$ using $PA = LU$ factorization with partial pivoting and two-step back substitution.

HW QUIZ 13

1. (10 points) Apply one step of Newton's Method with $x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to find x_1 the approximate solution to the system

$$\begin{cases} 4u^2 - 20u + \frac{1}{4}v^2 = -8 \\ \frac{1}{2}uv^2 + 2u - 5v + 8 = 0 \end{cases}$$

HW QUIZ 14

1. (5 points) Given the points $(1,1)$ and $(3,2)$. Use Newton's divided differences to find the interpolating polynomial of the points.

HW QUIZ 15 (with computer)

1. (5 points) Given the integral $\int_0^2 2x^3 \mathrm{d}x$.
   (a) Compute the composite Trapezoid rule with $m = 40$ and $m = 80$ intervals to approximate the integral using 4 decimal places. Compare with the exact value.
   (b) Verify the composite error formula $E = -\frac{(b-a)h^2}{12} f''(c)$ in your approximation in part (a).

HW QUIZ 16 (with computer)

1. (5 points) Use Euler's method and step size $h = 0.05$ to approximate $y(0.35)$ when y solves the IVP $y' = 4t - 2y, y(0) = 1$. Round to four decimal places.
2. (5 points) Compute the least squares solution and the 2−norm error of $\begin{bmatrix} 4 & 2 & 3 & 0 \\ -2 & 3 & -1 & 1 \\ 1 & 3 & -4 & 2 \\ 1 & 0 & 1 & -1 \\ 3 & 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 2 \\ 0 \\ 5 \end{bmatrix}$. Round all answers to four decimal places.