1. (a) Find the length of $a = (2, -2, 1)$ and find two independent vectors that are perpendicular to a.
   (b) What is the angle between $a = (2, -2, 1)$, and $b = (1, 2, 2)$?
   (c) What is the projection of $b = (1, 2, 2)$ onto $a = (2, -2, 1)$?
   (d) What is the projection of $b = (1, -2, 2)$ onto $a = (2, -2, 1)$? Verify $\hat{r} \perp a$.
   (e) What is the projection of $b = (1, 1, 1)$ onto the plane spanned by? $(1, 0, 0)$ and $(1, 1, 0)$ Verify $\hat{r} \perp (1, 0, 0)$ and $\hat{r} \perp (1, 1, 0)$.

2. Geometry of matrix multiplication as a linear transformation.
   (a) Project the vector $b = (1, 2, 2)$ onto the line through $a = (1, 1, 1)$. Check that $\hat{r} = b - Pb$ is perpendicular to a.
   (b) Find the best least squares solution $\hat{x}$ to $3x = 10, 4x = 5$. How is the residual minimized? Check that the residual $\hat{r} = b - A\hat{x}$ is perpendicular to the column of $A = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$.
   (c) Solve $Ax = b$ by least squares when $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$. Verify that the residual $\hat{r} = b - A\hat{x}$ is perpendicular to the columns of A.

3. Geometry of matrix multiplication as a linear transformation.
   (a) Project the vector $b = (1, 2, 8)$ onto the columns of $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$. Check that $\hat{r} = b - Pb$ is perpendicular to the columns of A.
   (b) Find the projection matrix $P = A(A^T A)^{-1} A^T$ when $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & -1 \end{bmatrix}$.
   (c) Redo the first question on this page using the projection matrix P.

4. (a) Find the least squares solution to Ax=b when $A = \begin{bmatrix} 6 & 9 \\ 3 & 8 \\ 2 & 10 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 49 \\ 0 \end{bmatrix}$. Then determine the $2-$norm of the residual.
   (b) Fit a linear function of the form $f(t) = b + mt$ to the data points $(0, 3), (1, 3)$, and $(1, 6)$.

5. Use Householder reflectors to find the QR factorization of the following matrices.
   (a) $A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$

6. (a) Find the orthogonal projection of $(9, 0, 0, 0)$ onto the subspace in 4 dimensional space spanned by $(2, 2, 1, 0)$ and $(-2, 2, 0, 1)$. ANSWER: $(8, 0, 2, -2)$.
   (b) Verify that the matrix $C = \frac{1}{2}\begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$ is orthogonal.
   (c) Write Python code to perform the Gram-Schmidt algorithm on the three columns a_1, a_2, and a_3 of the matrix $A = \begin{bmatrix} 2 & -2 & 18 \\ 2 & 1 & 0 \\ 1 & 2 & 0 \end{bmatrix}$

7. Find the Householder reflectors to find the QR factorization of $A = \begin{bmatrix} 1 & -4 \\ 2 & 3 \\ 2 & 2 \end{bmatrix}$. ANSWER: $H_1 = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$, $H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -.6 & -.8 \\ 0 & -.8 & .6 \end{bmatrix}$, and $QR = \frac{1}{3}\begin{bmatrix} 1 & -\frac{14}{5} & -\frac{2}{5} \\ 2 & 1 & -2 \\ 2 & 2 & 11 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 0 & 5 \\ 0 & 0 \end{bmatrix}$.

8. Textbook Exercises: $1, 2, 5, 7$.