

- Determine if each function has a unique fixed point in the given interval.
  - $g(x) = 1 - \frac{x^2}{4}$  on  $[0, 1]$ .
  - $g(x) = 2^{-x}$  on  $[0, 1]$ .
  - $g(x) = \frac{1}{x}$  on  $[0.5, 5.2]$ .
- Use FPI when  $g(x) = -4 + 4x - \frac{x^2}{2}$ . Find the actual errors and the actual relative errors for each iterate.
  - Find  $x_1, x_2, x_3$  when  $x_0 = 1.9$ .
  - Find  $x_1, x_2, x_3$  when  $x_0 = 3.8$ .
  - Use the FPI theorem to make conclude what will happen in these FPI sequences.
- Use FPI when  $g(x) = 0.5x + 1.5$ . Find the actual errors and the actual relative errors for each iterate.
  - Find  $x_1, x_2, x_3, x_4, x_5$  when  $x_0 = 4$ .
  - Find the fixed point (exactly) with your bare hands.
  - Can FPI be used to find a fixed point of  $g(x) = x^2 + x - 4$ . Explain.
- Make cobweb (staircase) diagrams for each of the following.
  - $g(x) = \sqrt{6 + x}, x_0 = 7$ .
  - $g(x) = 1 + \frac{2}{x}, x_0 = 4$ .
  - $g(x) = \frac{x^2}{3}, x_0 = 3.5$ .
  - $g(x) = -x^2 + 2x + 2, x_0 = 2.5$ .
- $f(x) = x \sin x$  has a root  $x^*$  between  $a = 2$  and  $b = 4$  because it is continuous and
  - Use the Bisection Method for  $f, a$  and  $b$  to find  $x_8$ .
  - Use the Bisection error formula to find an error bound for your approximate root  $x_8$ .
  - How many steps of Bisection are necessary to get 11 digits of accuracy after the decimal place?
- Let  $f(x) = e^x - x - 2$ .
  - Find  $a$  and  $b$  so that  $f(a) \cdot f(b) < 0$ .
  - Use the Bisection Method for  $f, a$  and  $b$  to find  $x_8$ .
  - Use the Bisection error formula to find an error bound for your approximate root  $x_{18}$ .
  - How many steps of Bisection are necessary to get 9 digits of accuracy after the decimal place?
- Use Newton's method to find the approximation  $x_3$  to  $x^* = \sqrt{5}$  when  $x_0 = 2$ . Give your answer with 12 decimal places of accuracy. Explain how you used Newton's.
  - Use the Secant Method to find the approximate roots  $x_2, x_3$ , and  $x_4$  to  $f(x) = x^3 - 3x + 2$  when  $x_0 = -2.6$  and  $x_1 = -2.4$ . Give your answers with four places of accuracy after the decimal.
- Use Newton's method to find the approximation  $x_{11}$  to a root of  $f(x) = x^3 - 3x - 2$  when  $x_0 = 2$ . Give your answer with 10 decimal places of accuracy.
  - Repeat the above problem with different initial approximations  $x_0 = -3, -2, -1, 0, 1$ , and then 3. Make a rough sketch of  $f(x) = x^3 - 3x - 2$  to explain the different convergence speeds.
- TEXTBOOK EXERCISES: 1, 3, 5, 6, 15.