9.1: Newton's Method for nonlinear systems

- $f_1(x_1, x_2) = x_1^2 - 2x_1 - x_2 + 1$ and $f_2(x_1, x_2) = x_1^2 + x_2^2 - 1$.
- Simultaneously solve: $f_1(x_1, x_2) = 0$ and $f_2(x_1, x_2) = 0$.
- This is a baby example found on p. 252 of which we already know the exact solutions $x^* = (1, 0)$ and $x^* = (0, 1)$. The purpose of this lesson is to find a robust algorithm to approximate these roots. We want an algorithm to approximate the roots of any nonlinear multivariable equation $f(x_1, x_2, \ldots, x_n) = 0$.

9.1: Newton's Method for nonlinear systems

- x_0 is the initial guess (x_0 is a vector).
- $x_{k+1} = x_k - J(x_k)^{-1} F(x_k)$.

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- x_0 is the initial guess (x_0 is a vector).
- $J(x_k)p_k = -F(x_k)$.
- $x_{k+1} = x_k + p_k$.

9.1 Example: Newton's Method for nonlinear systems

Solve the nonlinear system $0 = f_1(x_1, x_2) = x_1^2 - 2x_1 - x_2 + 1$ and $0 = f_2(x_1, x_2) = x_1^2 + x_2^2 - 1$.

- Verify that $0 = f_1(x_1, x_2)$ is a parabola and $0 = f_2(x_1, x_2)$ is a circle.
- Verify that $x^* = (1, 0)^T$ and $x^* = (0, 1)^T$ are only solutions.
- Use Newton's method to find $\vec{x}_1$ and $\vec{x}_2$ by hand when $\vec{x}_0 = (1, 1)^T$.

9.1 Example: Newton's Method Code for nonlinear systems

```
#ALGORITHM: Newton's Method for Systems p.254
import numpy as np

def Newton_method(F, DF, x, num_steps):
    """Applies Newton's Method FPI num_steps times
    to F with initial guess x0."""
    for k in range(num_steps):
        x = x - np.linalg.solve(DF(x),F(x))  # use numpy's
        print(k+1,x)
```

9.1 Example: Newton's Method Code for nonlinear systems

```
# Input to Newton's method:
# A function F(x); its Jacobian DF and
# an initial guess x = x0.
def F(x):
    """F is the multivariable function to study."""
    return np.array([x[0]**2 -2*x[0] - x[1] + 1, x[0]**2 +

def J(x):
    """J is the Jacobian of F."""
    return np.array([[2*x[0] - 2, -1],
                     [2*x[0], 2*x[1]]])

x = np.array([1,1]) # initial guess

Newton_method(F,J,x,7)
```

9.1: Newton's Method for nonlinear systems

- The multivariable Newton's code method itself is on the first slide. It is short and sweet, a one-line FPI just like Newton's method for nonlinear scalar equations in Chapter 3.
- The second slide indicates the particular nonlinear equation for Newton's method to approximate a solution. You can edit the second slide for different problems.