PART I. DO ALL 6 PROBLEMS (70 POINTS)

1. (8 points) Evaluate the limits:

(a) \( \lim_{x \to 0} \frac{\ln (1 + x)}{\sin (2x)} \)  
(b) \( \lim_{x \to 0^+} (2x)^{x^2} \)

2. (24 points) Integrate and simplify:

(a) \( \int x \ln x \, dx \)  
(b) \( \int \frac{x + 1}{x^2 + x - 6} \, dx \)  
(c) \( \int \cot^2 x \cos x \, dx \)  
(d) \( \int_0^2 \frac{dx}{x^2 + 4} \)

3. (15 points) Find the derivative \( \frac{dy}{dx} \) and simplify:

(a) \( y = \tan^{-1}(e^{x^2}) \)  
(b) \( y = x \sin x \)  
(c) \( \sinh(xy) + \ln(y^2) = 5 \cos x \)

4. (7 points) Sketch the graph of the polar equation \( r = 4 \sin (3\theta) \) (indicating relevant angles) and compute the area inside one loop.

5. (8 points) Find the length of the portion of the curve \( y = \frac{(x^4 + 3)}{6x} \) from \((1, \frac{2}{3})\) to \((3, \frac{14}{3})\).

6. (8 points) Find the volume of the solid generated by revolving about the \( y \)-axis the region enclosed by the line \( x = 1 \), and the graphs of \( y = \frac{1}{4}x^3 + 1 \) and \( y = 1 - x \).

PLEASE TURN OVER, EXAM CONTINUES
PART II. Do 3 COMPLETE PROBLEMS out of 5 (10 points each)

7. (a) Your ability to concentrate on this exam is decaying exponentially. After 1 hour your concentration is $1/4$ of what it was at the beginning of the exam. What fraction of your initial concentration will you have left after $3/2$ hours?

(b) Find the polynomial plus partial fractions decomposition of

$$\frac{x^3 + x^2}{x^2 + 5x + 6}$$

Note: for full credit you must solve for all coefficients.

8. (a) Use Simpson’s (parabolic) rule with 4 subintervals to approximate $\int_1^5 \frac{1}{x+1} \, dx$.

You may leave your answer as a sum of fractions.

(b) Decide which of the following is of indeterminate form and evaluate it. Explain why the other is not of indeterminate form:

(i) $\lim_{x \to \infty} \left( \frac{1}{x} \right)^x$

(ii) $\lim_{x \to -\infty} \left( 1 + \frac{1}{x} \right)^x$

9. A tank in the shape of a right circular cone of height 5 ft and radius 3 ft, with its circular base on the ground so that its axis is perpendicular to the ground and its vertex is pointing straight up into the air, is filled to a height of 2 ft with a liquid weighing 4 lb/ft$^3$. Find the work done in pumping the liquid out through the top vertex of the tank.

10. (a) Sketch the graph of $x^2 + 4y^2 - 2x + 16y + 1 = 0$, labeling the coordinates of the center, foci, vertices, and asymptotes, if any.

(b) Find the area of the region that lies between the curves $y = 4 - x^2$ and $y = x$.

11. (a) Find the value of the following improper integral, or show it diverges:

$$\int_0^1 \frac{x^2}{x^3 - 1} \, dx$$

(b) A curve is given parametrically by $x = t^2 - 2$, $y = t^3$, $-3 \leq t \leq 3$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $t \neq 0$, without eliminating the parameter $t$.

END OF EXAM