The City College Department of Mathematics
Fall 2012 Math 20200 Final Exam

PART I: Answer ALL questions in this part. (70 points)
PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 7): Answer all questions (70 points)

1) Find the derivative for each of the following (6 points each).
   a. \( y = \ln\left(\frac{x^3}{\sqrt{x+1}}\right) \)
   b. \( y = \arctan(x^2) \)
   c. \( y = x^x + \sin(x) \)

2) Compute each of the following integrals (6 points each).
   a. \( \int \frac{x^3 + 1}{x^3 + x} \, dx \)
   b. \( \int_0^{\pi/4} \tan^3(x) \sec(x) \, dx \)
   c. \( \int \frac{1}{x^4 \sqrt{x^2 - 4}} \, dx \)
   d. \( \int_1^2 x \ln(x) \, dx \)

3) Evaluate the limits (4 points each).
   a. \( \lim_{x \to \infty} \frac{(\ln(x))^2}{x} \)
   b. \( \lim_{x \to \pi} \frac{\cos(x)}{x^2 + 1} \)

4) (12 points) The region \( R \) in first quadrant of the \( xy \) plane is bounded by the curves \( y = \ln(x) \), \( y = 2 \) and \( x = 1 \). Set up two integrals (method of washers and method of shells) for the volume of the solid obtained by rotating \( R \) around the \( x \)-axis. Use one of these to compute the volume.
5) (8 points) Sketch the curve given by the equation $r = 1 + \cos(\theta)$ in polar coordinates, labeling the $x$ and $y$ intercepts, and compute the area it encloses.

END OF PART I

Part II: Answer 3 complete questions (10 points each)

6) A conical tank, supported with point down, has diameter 8 feet and height 10 feet. Assuming it contains water to a height of 7 feet of water (which has density 62.5 pounds per cubic foot), compute the work done pumping all the water out over the top.

7) 
   a. For the curve given parametrically by $x = t^2, y = t^3 - 3t$, compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
   b. For the graph of the function $y = \tan(x)$, set up an integral to compute the arc-length from $x = 0$ to $x = \pi/4$. Do not attempt to compute the integral.

8) 
   a. Evaluate the integral or show it is divergent: $\int_0^{\infty} te^{-t} \, dt$.
   b. Write out the form of the partial fraction decomposition of the following function. Do not attempt to determine the numerical values of the coefficients.
   
   $f(x) = \frac{x^2 + 1}{x(x - 1)^3(x^2 + 9)}$

9) 
   a. Draw a sketch of the conic whose equation is $y^2 + 2y = 9x^2 + 35$. Identify which sort of conic it is. On your sketch, show and label whichever of the following are present: vertices, asymptotes, and foci.
   b. Use the definitions of the hyperbolic trig functions to derive the identity: $\cosh^2(t) - \sinh^2(t) = 1$. 
10) 

a. A radioactive substance has a half-life of 15 years. If you begin with 400 pounds of the substance, how much will be left after 60 years?

b. Sketch the curves $y = x^3$ and $y = 4x$. Compute the area of the entire bounded region which has these two curves as boundaries.