The City College Department of Mathematics
Fall 2010 Math 20100  Final Exam

CALCULATORS are NOT allowed.

PART I: Answer ALL questions in this part. (70 points)
PART II: Answer three complete questions out of five. Each question is worth 10 points. If you answer more than three questions, cross out work you do not want graded.

Part 1 (questions 1 to 7): Answer all questions (70 points)

1) Find \( \frac{dy}{dx} \) (explicitly or implicitly) and simplify where possible (5 points each).
   a) \( y = \cos^3(2x) \)
   b) \( y = x^3(3x - 1)^5 \)
   c) \( y = \frac{x}{(2x + 5)^3} \)
   d) \( x \sin y = 1 - x^2y \)

2) Find each integral (5 points each).
   a) \( \int (4x - 1 + 3\sqrt{x}) \, dx \)
   b) \( \int \frac{x}{(x^2 + 4)^5} \, dx \)
   c) \( \int_{\frac{\pi}{6}} \sin(\pi x^2) \, dx \)
   d) \( \int \frac{(x - 6)^2}{x^4} \, dx \)

3) (6 points) A particle moving in a straight line has an acceleration given by \( a(t) = 2t \). The initial velocity of the particle is 2 cm/sec. How far does the particle move between \( t = 1 \) and \( t = 2 \) seconds?

4) (6 points) Find all the asymptotes, intercepts, maximum, minimum, and inflection points for \( y = \frac{4x}{x^2 + 1} \), and sketch the graph, labeling such points. For your information \( y' = \frac{4(1-x^2)}{(x^2+1)^2} \) and \( y'' = \frac{8x(x^2-3)}{(x^2+1)^3} \).

5) (6 points) Find a) \( \lim_{x \to 2} \frac{\sqrt{x+2} - 2}{x-2} \)
     b) \( \lim_{t \to 0} \frac{(1 + \cos 2t)(1 - \cos 2t)}{t^2} \)

6) (6 points) Find the maximum and minimum values of \( f(x) = x^3 + 3x^2 \) on the interval \([-3, 2]\).
7) (6 points) Let \( g(x) = 3 + \frac{2}{x} \). Using the definition of derivative (no other method allowed), find \( g'(x) \).

END OF PART I

Part II: Answer 3 complete questions (10 points each)

8) You have $1200 to buy fencing, which costs $10 per horizontal foot. The fence will surround a rectangular field and split the field down the middle, as shown in the figure to the right. What are the dimensions that will enclose the maximum possible area?

9) The picture shows car A traveling southward towards point P at 45 mi/hr. Car B is traveling east away from P at 30 mi/hr. At the instant when the distance AP is 60 mi and PB is 80 mi, what is the rate of change of distance AB?

10) a) Let \( F(x) = \int_0^x \sqrt{t^2 + 9} \, dt \). Evaluate \( F''(4) \).

b) Given \( h(x) = x^{\frac{1}{3}} \). Find the exact value of \( c \) in the interval \( (0,1) \) guaranteed by the Mean Value Theorem applied to \( h(x) \).

11) a) Use differentials to estimate \( \sqrt{3.99} \).

b) Find a point on the graph of \( y = x^3 - 3x \) where the tangent line is parallel to the line \( y = 9x - 8 \), and write the equation of the tangent line at the point.

12) Define \( f(x) = \begin{cases} 
  x + 2 & \text{if } x < 3 \\
  8 - x & \text{if } 3 \leq x < 5 \\
  4 & \text{if } x = 5 \\
  3 & \text{if } x > 5 
\end{cases} \)
a) Sketch the graph of \( f(x) \) for \( 0 \leq x \leq 7 \)
b) Find each of the following limits or explain why the limit doesn't exist.
   i) \( \lim_{{x \to 3}} f(x) \)
   ii) \( \lim_{{x \to 4}} f(x) \)
   iii) \( \lim_{{x \to 5}} f(x) \)
c) Is \( f(x) \) continuous at \( x = 5 \)? Explain.