

Exercise 8.1 Using contour, plot at least **10 level curves** of the function $f(x, y) = x^2 - y^2$ on the rectangle $\mathbf{R} = \{(x, y) \mid -3 \leq x \leq 3, -3 \leq y \leq 3\}$. Add a plot of the gradient field to the plot of the level curves. Make sure you adjust the scales using axis equal to preserve orthogonality. Publish your M-file with the graphic output.

```
clear all
clc
figure(1)

syms x y

f=@(x,y)x.^2 - y.^2;

fx=@(x,y)2.*x;
fy=@(x,y)-2.*y;

x=linspace(-3,3,20);
y=linspace(-3,3,20);

[X,Y] = meshgrid(x,y);

contour(X,Y,f(X,Y),12,'k','LineWidth',2)

hold on

quiver(X,Y,fx(X,Y),fy(X,Y),'b')

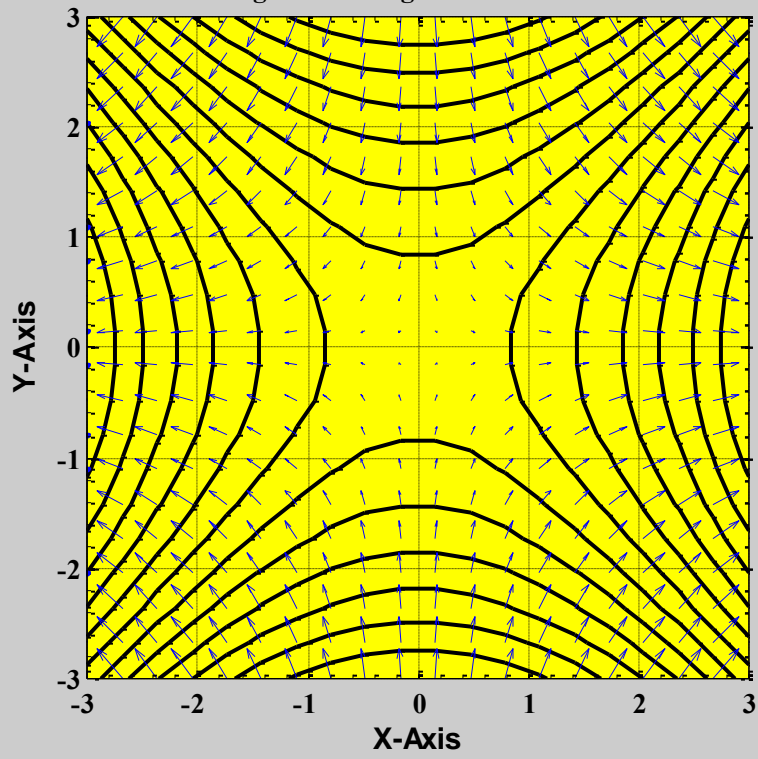
rotate3d
grid on
axis equal square
axis([-3 3 -3 3])

set(gca,'Color','y')
set(gca,'XTick',-3:1:3,'XMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'YTick',-3:1:3,'YMinorTick','on','FontName','times','FontWeight','bold')

title({'12 Level Curves of f(x,y) = x^2-y^2','together with gradient its field'})

xlabel('X-
      Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize',12)
ylabel('Y-
      Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize',12)
```

12 Level Curves of $f(x,y) = x^2 - y^2$
together with gradient its field



Exercise 8.2 Using the parametric representation $x = \cosh(t)$, $y = \sinh(t)$, plot the curve $x^2 - y^2 = 1$. Include portions of both branches of this hyperbola. Then add the normal vectors to the plot of the curve. (Your plot should only show arrows attached to the hyperbola, not covering a 2D region.) Make sure you adjust the scales using **axis equal** to preserve orthogonality. Publish your M-file, including the graphic output.

```
figure(2)
syms x y t
t=linspace(-3,3,40);

x = cosh(t);
y = sinh(t);

plot(x,y,-x,y,'b','LineWidth',2)

hold on

quiver(x,y,2*x,-2*y,'r')

quiver(-x,y,-2*x,-2*y)

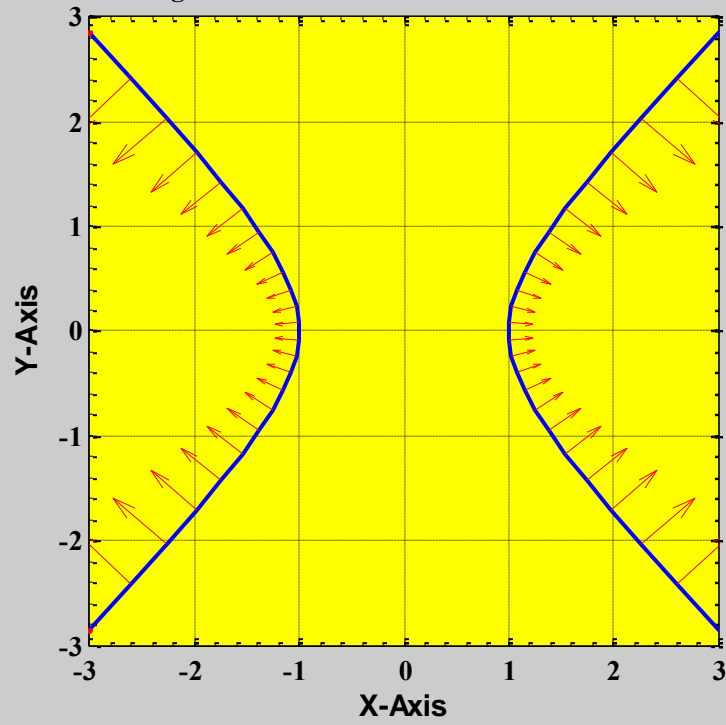
grid on
axis equal square
axis([-3 3 -3 3])

set(gca,'Color','y','XTick',-
    3:1:3,'XMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'YTick',-3:1:3,'YMinorTick','on','FontName','times','FontWeight','bold')

title({' ','Graph of x^2 - y^2 = 1','together with vectors normal to the curve'})

xlabel('X-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
ylabel('Y-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
```

Graph of $x^2 - y^2 = 1$
together with vectors normal to the curve



Exercise 8.3 Plot the surface $x^2 + 3y^2 + 4z^2 = 9$ by **parametrizing it** (see Chapter 7 on parametric plotting for how to do this using spherical coordinates). On the same graph plot the normal vector field to this surface. Make sure you adjust the scales using **axis equal** to preserve orthogonality. Publish your M-file, including graphic output.

```
figure(3)

u = linspace(0,pi,35);
v = linspace(0,2*pi,35);

[U,V] = meshgrid(u,v);

X = 3.*sin(U).*cos(V);
Y = sqrt(3).*sin(U).*sin(V);
Z = (3/2).*cos(U);
VX = 2.*X;
VY = 6.*Y;
VZ = 8.*Z;
surf(X,Y,Z)

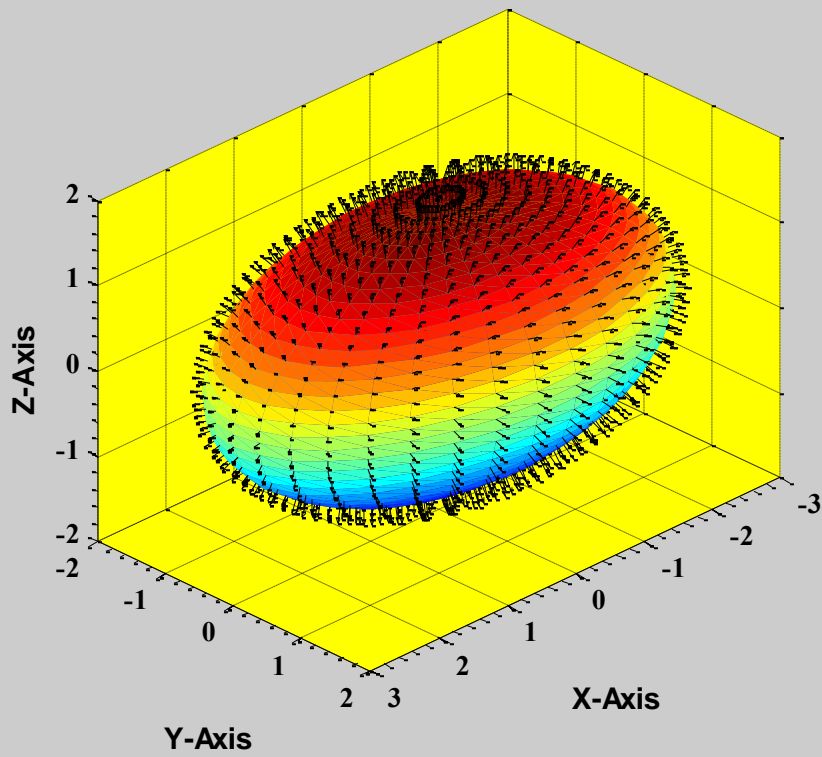
shading interp
grid on
hold on
quiver3(X,Y,Z,VX,VY,VZ,'k')
rotate3d
view([135,28])
axis equal
axis([-3,3 -2 2 -2 2])

set(gca,'Color','y','XTick',-
    3:1:3,'XMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'YTick',-2:1:2,'YMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'ZTick',-2:1:2,'ZMinorTick','on','FontName','times','FontWeight','bold')

title({'Graph of the parametrized ellipsoid  $x^2 + 3y^2 + 4z^2 = 9$ ','together with
    outward normal vectors'})

xlabel('X-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
ylabel('Y-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
zlabel('Z-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
```

Graph of the parametrized ellipsoid $x^2 + 3y^2 + 4z^2 = 9$
together with outward normal vectors



Exercise 8.4 Recreate the picture in Example 8.4 but using the **sphere** command to generate the points on the sphere.

```
figure(4)

n=35;
[x,y,z] = sphere(n);

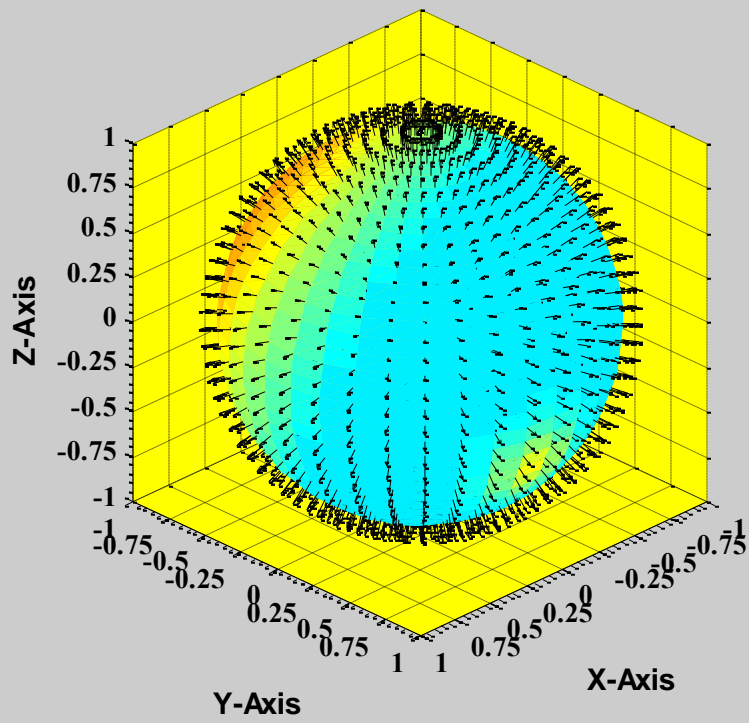
surf1(x,y,z)
shading interp
hold on
grid on
quiver3(x,y,z,2*x,2*y,2*z,'k')
rotate3d
view([135,28])
axis equal
axis([-1 1 -1 1 -1 1])

set(gca,'Color','y','XTick',-
    1:0.25:1,'XMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'YTick',-1:0.25:1,'YMinorTick','on','FontName','times','FontWeight','bold')
set(gca,'ZTick',-1:0.25:1,'ZMinorTick','on','FontName','times','FontWeight','bold')

title({'Graph of the unit sphere  $x^2 + y^2 + z^2 = 1$ ','together with outward normal
    vectors','using the sphere command'})

xlabel('X-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
ylabel('Y-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
zlabel('Z-
    Axis','Color','black','FontName','mathematica','FontWeight','bold','FontSize'
    ,12)
```

Graph of the unit sphere $x^2 + y^2 + z^2 = 1$
together with outward normal vectors
using the sphere command



Exercise 8.5 Matlab has a **gradient** command that approximates a gradient numerically. Look up how to use the command and then use it to generate the output in Example 8.3.

```
figure(5)
f = @(x,y)x.^2+y.^2;
F = @(x,y,z) x.^2+y.^2-z;
x = linspace(-2,2,25);
y = linspace(-2,2,25);
z = linspace(0,8,25);

[X,Y] = meshgrid(x,y);
[U,V,W]=meshgrid(x,y,z);
surf(X,Y,f(X,Y));
hold on

shading interp

[FX,FY,FZ]= gradient(F(U,V,W),0.2,0.2,0.2);
quiver3(U,V,f(U,V),FX,FY,FZ, 'k'); % black arrows
grid on
view([1 1 1])
axis equal
axis([-2,2 -2 2 0 8])

set(gca, 'Color', 'y', 'XTick', -
    2:1:2, 'XMinorTick', 'on', 'FontName', 'times', 'FontWeight', 'bold')
set(gca, 'YTick', -2:1:2, 'YMinorTick', 'on', 'FontName', 'times', 'FontWeight', 'bold')
set(gca, 'ZTick', 0:1:8, 'ZMinorTick', 'on', 'FontName', 'times', 'FontWeight', 'bold')

title({'Graph of  $z = x^2 + y^2$ ', 'together with outward normal vectors', 'using the
    gradient command'})

xlabel('X-
    Axis', 'Color', 'black', 'FontName', 'mathematica', 'FontWeight', 'bold', 'FontSize'
    ,12)
ylabel('Y-
    Axis', 'Color', 'black', 'FontName', 'mathematica', 'FontWeight', 'bold', 'FontSize'
    ,12)
zlabel('Z-
    Axis', 'Color', 'black', 'FontName', 'mathematica', 'FontWeight', 'bold', 'FontSize'
    ,12)
```

Graph of $z = x^2 + y^2$
together with outward normal vectors
using the gradient command

