1a: The direction vectors are scalar multiples, so the lines are parallel.

1b: We use cross product of a direction vector and a displacement vector from a point on one line to the other to get a normal to the plane:

\[
\text{Cross}\left\{\{1, -3, 4\}, \{2, 0, -1\}\right\} - \{5, 1, 1\}\right\} = \{10, -10, -10\}
\]

So an equation of the plane is \(10(x-2)-10(y-0)-10(z+1)=0\) or more simply, after dividing by 10, \((x-2)-(y) - (z+1)) = 0\)

2a: We take the gradient

\[
f = 10 + \frac{25}{z^2 + 1} \sin\left[2 \cdot x^2 + y^3 + z\right]
\]

\[
\text{grad} f = \{D[f, x], D[f, y], D[f, z]\}
\]

\[
\{4 \cdot \cos\left[2 \cdot x^2 + y^3 + z\right], 3 \cdot y^2 \cos\left[2 \cdot x^2 + y^3 + z\right], -\frac{50z}{(1+z^2)^2} + \cos\left[2 \cdot x^2 + y^3 + z\right]\}
\]

evaluated at \((1,0,-2)\) gives:

\[
\text{grad} f = \{D[f, x], D[f, y], D[f, z]\} / \{x \rightarrow 1, y \rightarrow 0, z \rightarrow -2\}
\]

\{(4, 0, 5)\}

The directional derivative is obtained by dotting the gradient with a unit vector in the desired direction:

\[
u = \{1, 2, 5\} \cdot \text{grad} f
\]

\[
\frac{29}{\sqrt{30}}
\]

2b: We solve to find that the point of interest is when \(t=1\) we use the chain rule to get

\[
\frac{dP}{dt} = \frac{\partial P}{\partial x} \frac{dx}{dt} + \frac{\partial P}{\partial y} \frac{dy}{dt} + \frac{\partial P}{\partial z} \frac{dz}{dt}, \text{ which gives}
\]

\[
r = \{2t - 1, \exp\{2t - 2\} - 1, t^3 - t - 2\}
\]

\[
\{-1 + 2t, -1 + e^{2t-2}, -2 - t + t^3\}
\]

\[\text{Solve}[r = \{1, 0, -2\}]
\]

\[\{(t \rightarrow 1)\}\]

\[
\text{D}[r, t]
\]

\[\{2, 2 e^{-2t}, -1 + 3t^2\}\]
\[D[r, t] \rightarrow 1\]
\[
\begin{align*}
(2, 2, 2)
\end{align*}
\]
\[\text{grad } f = \{D[f, x], D[f, y], D[f, z]\} \rightarrow \{x \rightarrow 1, y \rightarrow 0, z \rightarrow -2\}\]
\[
\begin{align*}
(4, 0, 5)
\end{align*}
\]
\[dPdt = 2 \times 4 + 2 \times 0 + 2 \times 5\]
\[18\]
3: Taking the first partials and setting them to zero and solving gives:
\[f = -3y^3 - 4x^2 + 8x + 9y\]
\[8x - 4x^2 + 9y - 3y^3\]
\[
\text{Solve}\left\{\{D[f, x] = 0, D[f, y] = 0\}\right\}
\[
\begin{align*}
\{\{x \rightarrow 1, y \rightarrow -1\}, \{x \rightarrow 1, y \rightarrow 1\}\}
\end{align*}
\]
At the first point, we evaluate the discriminant to get:
\[D[f, x, x]D[f, y, y] - D[f, x, y]^2 \rightarrow \{x \rightarrow 1, y \rightarrow -1\}\]
\[-144\]
So there is a saddle there.
At the second point, we evaluate the discriminant to get:
\[D[f, x, x]D[f, y, y] - D[f, x, y]^2 \rightarrow \{x \rightarrow 1, y \rightarrow 1\}\]
\[144\]
Since it is positive, we look at \(f_{xx}\):
\[D[f, x, x] \rightarrow \{x \rightarrow 1, y \rightarrow 1\}\]
\[
\begin{align*}
\{-8\}
\end{align*}
\]
So there is a relative max at (1,1).
4a: This region is described by the region:

\[
\begin{align*}
\text{Show[}&\text{Plot[}\sqrt{x}, \{x, 0, 1\}, \text{PlotRange} \to \{0, 2\}\],} \\
&\text{Plot[}2-x, \{x, 1, 2\}, \text{PlotRange} \to \{0, 4\}\]}
\end{align*}
\]

This region could be broken up into two parts to get:

\[
\int_0^1 \int_0^{\sqrt{x}} y \, dy \, dx + \int_1^2 \int_0^{2-x} y \, dy \, dx
\]

Or could be done x-integral first to get parts to get:
\[
\int_0^1 \int_{y^2}^{2-y} x \, dx \, dy
\]

16
15

4b. Use nearby point (3, 4) and adjust:

\[
f = \frac{25}{x^2 + y^2}
\]

\[
\frac{25}{x^2 + y^2}
\]

\[
df/dx = D[f, x] \quad \{x \to 3, \ y \to 4\}
\]

- \frac{6}{25}

\[
df/dx = D[f, y] \quad \{x \to 3, \ y \to 4\}
\]

- \frac{8}{25}

approx \approx \approx 5 + \frac{D[f, x] \quad \{x \to 3, \ y \to 4\}}{1} + \frac{D[f, y] \quad \{x \to 3, \ y \to 4\}}{0.1} - 0.2

4.456

5a: In polar we get for the volume after finding the intersection to be a circle of radius 2, with the first surface being the lower one and the second one the top one, we integrate the top - bottom to get:

\[
\int_0^{2\pi} \int_0^2 (8 - r^2 - r^2) \, r \, dr \, d\theta
\]

16 \pi

5b: We rewrite to get an implicit description, use the gradient and evaluate to get:

\[
f = x \, y^2 - \log(2 \, z - 1)
\]

\[
x \, y^2 - \log(-1 + 2 \, z)
\]

\[
D[f, x] \quad \{x \to 2, \ y \to -1, \ z \to 1\}
\]

1

\[
D[f, y] \quad \{x \to 2, \ y \to -1, \ z \to 1\}
\]

-4

\[
D[f, z] \quad \{x \to 2, \ y \to -1, \ z \to 1\}
\]

-2

which gives an equation of the tangent plane as

\[
1 \, (x - 2) + -4 \, (y + 1) + -2 \, (z - 1) = 0
\]

6a: Diverges by the test for divergence.

6b: Converges absolutely by the ratio test

6c: Conditionally convergent by: 1) alternating series test 2) integral test
7: We use the ratio test to get convergence on the interval \(-3 < x < -1\). For \(x = -1\), divergent by comparison with the harmonic series. For \(x = -3\), convergent by the alternating series test. So the power series converges on the interval \((-6, 2]\).

\[
a[n_] := \frac{(n + 1) (x + 2)^n}{(n + 2)^2}
\]

\[
\text{Limit}[\text{Abs}\left[\frac{a[n + 1]}{a[n]}\right], n \to \text{Infinity}]
\]

\[
\text{Abs}[2 + x]
\]

7b: Approaching along the x-axis gives a limit of 1, and approaching along the y-axis gives a limit of 0, so the limit does not exist.

Alternatively, we can use polar and cancel out \(r^2\) from numerator and denominator to get the numerator is \(\cos^2(\theta)\) whose value depends upon which direction is approached, so the limit does not exist.

8a: differentiate to find the tangent vector at the relevant time \((t=0)\)

\[
r = \left\{\sqrt{t + \text{Exp}[t]}, 2t + 5, t^3 + 2\right\}
\]

\[
\left\{\sqrt{e^t + t}, 5 + 2t, 2 + t^3\right\}
\]

\[
\text{D}[r, t] /. t \to 0
\]

\[
\{1, 2, 0\}
\]

giving the unit vector after dividing by the length \(\sqrt{5}\)

8b: rearrange to put into standard form of \(\frac{(x - 3)^2}{2^2} - \frac{y^2}{1^2} - \frac{z^2}{1^2} = 1\) gives a hyperboloid of two sheets opening up in the +x directions, with vertices at \((5,0,0)\) and \((1,0,0)\).

9a: We use cylindrical coordinates to find the mass:

\[
\text{mass} = \int_0^{2\pi} \int_0^1 \int_0^{r^2} 2z \, r \, dz \, dr \, d\theta
\]

\[
\frac{\pi}{4}
\]

9b: We use spherical coordinates to find the mass:

\[
\text{mass} = \int_0^{2\pi} \int_0^{\pi/4} \int_0^{\rho} \rho \cos[\phi] \rho^2 \sin[\phi] \, d\rho \, d\phi \, d\theta
\]

\[
\frac{2\pi}{15}
\]

10a: The series is obtained from known series by substitution and multiplying:

\[
\text{Series}[\text{Exp}[-x^4], \{x, 0, 14\}]
\]

\[
1 - x^4 + \frac{x^8}{2} - \frac{x^{12}}{6} + 0[x]^{15}
\]

10b: We evaluate the definite integral to get:
\[ \int_0^1 (1 - x^4) \, dx \]
\[ 11.381 \]
\[ 23.04 \]

Since the series is alternating, the error is less than the next term, which is smaller than \( \frac{x^9}{9} \left( \frac{1}{2} \right)^9 \) and since \( 2^9 \) is 512 which when multiplied by 9 is more than 1000, the error is less than \( \frac{1}{1000} \).

10b: Integrate over the shadow to get possibly:

\[
\text{area} = \int_0^1 \int_0^\infty \frac{\sqrt{1 + \theta^2 + (2 \, y)^2}}{\theta^2 + (2 \, y)^2} \, dy \, dx
\]
\[ \frac{1}{12} \left( 1 + \sqrt{5} + 3 \, \text{ArcSinh}[2] \right) \]

Easier with respect to \( x \) first to get the integral manageable via substitution:

\[
\text{area} = \int_0^1 \int_0^\infty \frac{\sqrt{1 + \theta^2 + (2 \, y)^2}}{\theta^2 + (2 \, y)^2} \, dx \, dy
\]
\[ \frac{1}{12} \left( -1 + 5 \, \sqrt{5} \right) \]