Sample Final 2, Fall 2018
Math 201

Instructions: Please read each question carefully, show all work, and check afterwards that you have answered all of each question correctly. **Important: No books, calculators, cell phones, computers, laptops or notes are allowed.** You must show **all** your work to receive credit. Any crossed out work will be disregarded (even if correct). Write **one** clear answer with a coherent derivation for each question. You have 135 minutes to complete this exam. Good luck!

[1] **(16 pts, 4 pts each)** Compute the derivative $f'(x)$ for each of the functions below. You do not need to simplify your answer.

(a) $f(x) = \sin(x^2)\cos^4(\sqrt{x})$

(b) $f(x) = (\ln x)\,(\sqrt[3]{x} - \dfrac{1}{x^2})$

(d) $f(x) = \dfrac{\arcsin(x)}{e^{4x}}$

(e) $f(x) = x^{x+3}$

[**1**] (16 pts)

Please leave blank!

[2] **(16 pts, 4 pts each)** Find each integral and simplify your answer.

(a) $\displaystyle\int_1^e \frac{\ln(x)}{x}\,dx$

(b) $\displaystyle\int \sin^3(3x)\cos(3x)\,dx$

(c) $\displaystyle\int \frac{36dx}{(2x+1)^3}$

(d) $\displaystyle\int x^2 2^{x^3}\,dx$

[**2**] (16 pts)

Please leave blank!

[3] (9 pts, 3 pts each) Find the limits, or state that the limit does not exist (you must justify your answer):

(a) $\lim_{x \to +\infty} \left(\sqrt{x^2 + 2x} - \sqrt{x^2 - x} \right)$

(b) $\lim_{x \to 0^+} (\ln x - \ln(\sin x))$

(c) $\lim_{x \to +\infty} x^{1/(\ln x)}$

[3] (9 pts)

Please leave blank!

[4] **(6 pts, 3 pts each)**

(a) Using the limit definition of derivative, find $f'(x)$ for $f(x) = \sqrt{x}$. No credit will be given for any other method.

(b) Find an equation of the tangent line to the graph $y = \sqrt{x}$ at the point where $x = 9$.

[4] (6 pts)

Please leave blank!

[5] (8 pts, 4 pts each)

(a) Let $F(x) = \int\limits_0^{\tan(x)} \sqrt{1-t^3}\,dt$. Find $F'(x)$.

(b) Find $\dfrac{dy}{dx}$ for the curve $e^{2x} = \sin(x+3y)$.

[5] (8 pts)

Please leave blank!

[6] (8 pts, 4 pts each)

(a) A cube's surface area increases at the rate of $72\,\text{in}^2/\text{sec}$. At what rate is the cube's volume changing when the edge length is $3\,\text{in}$? Be sure to include units in your answer.

(b) Find the absolute maximum and minimum values of the function $f(x)$ for x in the interval $[-1, 3]$, where f is given by $f(x) = \begin{cases} x^2 & \text{if } -1 \leq x < 2, \\ 8 - 2x & \text{if } 2 \leq x \leq 3. \end{cases}$

[6] (8 pts)

Please leave blank!

[7] (5 pts) Use differentials to estimate $\sin(\pi/6 + 0.01)$. You do not need to simplify your answer.

[7] (5 pts)

Please leave blank!

[8] **(8 pts)**

(a) (2 pts) State the Intermediate Value Theorem, including the hypotheses.

(b) (2 pts) State the Mean Value Theorem, including the hypotheses.

(c) (4 pts) Prove that the function $f(x) = 2x^7 - 1$ has **exactly one** real root in the interval $[0, 1]$. More precisely, show that there is **at least** one real root, and **at most** one real root.

[**8**] (8 pts)

Please leave blank!

[9] (8 pts)

(a) (4 pts) Let $f(x) = \begin{cases} (x-1)^2 & \text{if } x < 1, \\ ax+b & \text{if } 1 \leq x \leq 4, \\ \sqrt{2x+1} & \text{if } x > 4. \end{cases}$

Find a and b so that $f(x)$ is continuous for all x, and justify your answer.

(b) (4 pts) Using Riemann Sums, write down an expression that estimates $\int_0^2 x^2 dx$ using the Left Endpoint Rule with 5 subdivisions. You may leave your answer as a sum of unsimplified fractions.

[9] (8 pts)

Please leave blank!

[10] (8 pts) A rectangular box has a square base and an open top. The four sides are made of wood that costs 2 dollars per square foot, while the base is made of aluminum that costs 25 dollars per square foot. If the volume of the box is to be 50 cubic feet, what is its minimum possible cost?

Note: Justify your answer using calculus.

[10] (8 pts)

Please leave blank!

[11] (8 pts) For the function $f(x) = \dfrac{x^2}{x^2-1}$, you are given (do not compute!) that

$$f'(x) = \frac{-2x}{(x^2-1)^2} \quad \text{and} \quad f''(x) = \frac{2(3x^2+1)}{(x^2-1)^3}.$$

(a) Find the domain of $f(x)$.

(b) Find the coordinates of all intercepts, and the equations of all asymptotes, of the graph of $y = f(x)$.

(c) In what intervals is the function f increasing? decreasing?

(d) In what intervals is the graph of f concave up? concave down?

(e) Find the coordinates of all local maxima, local minima, and points of inflection of the function f.

(f) Sketch the graph of $y = f(x)$. Label the features you found in items b and e.

[11] (8 pts)

Please leave blank!