1 Do five of the following six problems: 1–6

1. Prove the trigonometric identity \( \frac{\sin 2x}{1 + \cos 2x} = \tan x \).

2. Simplify \( \frac{x+3}{x^2+y^2} \div \frac{x^2+7x+12}{x^2+y^2-15} \).

3. Rewrite without parentheses or a radical sign \( y^3 \left( \sqrt{y} - \frac{1}{\sqrt{y}} \right) \).

4. Solve \( \frac{5}{x+2} - \frac{6}{x} + 1 = 0 \) for \( x \).

5. Find all \( x \) that solve \( x^3 > x^2 \).

6. Given \( f(x) = 2x - 4x^2 \). Find and simplify
   
   (a) \( f(3-a) \)
   
   (b) \( f\left(\frac{a}{2}\right) \)

2 Do five of the following six problems: 7–12

7. Show that the equation \( x^2 + y^2 + \frac{1}{2}x - 2y + \frac{1}{16} = 0 \) represents a circle, and find its center and radius.

8. Given \( f(t) = t - 2t^2 \). Find and simplify the average rate of change of \( f \) from \( t = 2 \) to \( t = 2 + h \).

9. Sketch the graph of the function \( y = 2 - \sqrt{x+1} \). Label at least three points on your graph including any intercepts. Begin with \( y = \sqrt{x} \) and indicate the steps needed to transform its graph to the graph of \( y = 2 - \sqrt{x+1} \).

10. Given \( f(x) = \frac{x+2}{x+1} \) and \( g(x) = \frac{2}{x} \). Evaluate and simplify \( f(g(7)) - g(f(7)) \).

11. Let

    \[
    f(x) = \begin{cases} 
    1 - 2x & \text{if } x \leq 1, \\
    2x & \text{if } x > 1 
    \end{cases}
    \]

   (a) Evaluate \( f(-2) \) and \( f(1) \).

   (b) Sketch the graph of \( f(x) \) from \( x = -3 \) to \( x = 3 \).

12. Sketch the graph of \( p(x) = -x(x+2)^2(x-2)^3 \). Label all intercepts and indicate the end behavior.

3 Do five of the following six problems: 13–18

13. A bacteria culture starts with 900 bacteria. After one hour the count is 1000.

   (a) Assuming that bacteria population grows exponentially, find a function that models the number of bacteria \( n(t) \) after \( t \) hours. (You may leave \( e, \log, \) or \( \ln \) in your answer).

   (b) After how many hours will the number of bacteria double? (You may leave \( e, \log, \) or \( \ln \) in your answer).

14. Given the function \( f(x) = 2 - x^3 \).

   (a) Sketch the graph of \( f \).

   (b) Find the formula for the inverse function \( f^{-1}(x) \).

15. For the graph of the function in Figure 1, determine the interval(s) on which the function is increasing.

16. Solve for \( t \) in each of the following parts. You may leave \( e, \log, \) or \( \ln \) in your answer.
Figure 1:

(a) \(5e^{2t} - 20 = 0\)
(b) \(3 - \log(3 - x) = 1\).

17. Evaluate \(\log_3\left(\frac{1}{\sqrt[3]{27}}\right)\).

18. Perform the subtraction \(\frac{7}{x+6} - \frac{1}{x^2+8x+12}\) and simplify.

4 Do five of the following six problems: 19–24

19. Find the exact value of \(\tan(-\frac{20\pi}{3})\).

20. For \(y = -3\sin(x + \frac{\pi}{3})\) find the amplitude, period, phase shift and then graph. Label the coordinates of three points on your graph: one maximum point, one minimum point and one intercept.

21. Find \(\cos 105^\circ\).

22. Find all solutions \(\theta\) to \(6\cos^2\theta - 3 = 0\) for \(\theta\) in the interval \(-\pi \leq \theta \leq \pi\).

23. Evaluate each of the following
   (a) \(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\)
   (b) \(\arcsin(\sin(\frac{11\pi}{6}))\)

24. Find the radius \(r\) of the circle in Figure 2 with shaded angle of measure \(\frac{1}{2}\) radians that is subtended by the arc of length 6.
Figure 2: