MATHEMATICS 205 **FINAL EXAM** **spring, 2014**

NAME ____________________________

SECTION______________

INSTRUCTOR____________________________

(1) Turn off all cell phones and put them and all notes away and out of sight.

(2) NO CALCULATORS. NO scrap paper. Use the paper provided.

(3) Leave all numerical answers in exact form. Simplify when reasonable but leave the answers in terms of π, $\sqrt{}$, e, ln and fractions.

(4) SHOW YOUR WORK. Points will be deducted for solutions which are not supported by your written work.

Part I Answer all questions in this part (60 POINTS).

(1) Find $\frac{dy}{dx}$ and simplify when reasonable. (5 points each):

(a) $y = \frac{\ln(x)}{x+1}$

(b) $y = (e^{3x} + x^2)^5$

(c) $y = x(x^2 + \ln(3x^2))$.

(d) $y = 2^{5x}$.

(2) Compute each of the following integrals. (5 points each):

(a) $\int 1+\frac{2x+1}{x^2+x+1}\ dx$

(b) $\int \frac{x^3+3x^2+2}{x}\ dx$

(c) $\int (2x-1)(x^2-1)^2\ dx$

(d) $\int_1^e \frac{\ln(x^4)}{x}\ dx$

(3) For the curve given by the equation $y = x^3 + \frac{3}{2}x^2 - 6x$

(a)(8 points) Sketch the graph, labeling the relative maxima (MAX), relative minima (MIN) and points of inflection (POI). Describe where the curve is increasing and where decreasing. Describe where the curve is concave up and where concave down.

(b) (8 points) Determine the (absolute) maximum and minimum values for $f(x) = x^3 + \frac{3}{2}x^2 - 6x$ on the closed interval $[-1, 2]$.

(4) (4 points) A bacterial culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 420. Find an expression for the population size after t hours <u>and</u> compute when the population will reach $10,000$.

PartII Answer all sections of four (4) questions out of the six (6) questions in this part (10 points each).

(5) A function f satisfies the conditions: $f(0) = 6, f(4) = 1$, $f'(0) = f'(4) = 0$, and $f''(x) < 0$ when $x < 1$ while $f''(x) > 0$ when $x > 1$.

(a) Where is the derivative $f'(x)$ negative?

(b) Sketch a graph for $y = f(x)$ which is consistent with the given information.

(6) Two cars start moving from the same point. One travels south at 60 miles per hour and the other travels east at 25 miles per hour. At what rate is the distance between the cars increasing 2 hours later?

(7) (a) Compute the limits:

(i) $Lim_{x\to 1} \frac{x^2 + x - 6}{x^2 - x - 2}$

(ii) $Lim_{x\to -3} \frac{x^2 - 9}{x^2 + x - 6}$.

(b) Compute the area of the bounded region between the curves $y = 4 - x^2$ and $y = x^2$.

(8) (a) Assume that the function f is differentiable and satisfies $f(20) = 200$ and $f'(20) = -2$. Use linear approximation to estimate $f(23)$.

(b) Use the definition of the derivative to compute the derivative of the function given by $f(x) = \frac{1}{x+1}$ and show that you answer agrees with the one given by the differentiation rules.

(9) A rectangular box with an open top is to have volume 10 cubic feet. The length of the base is to be twice the width. Material for the base costs \$10 per square foot and for the sides costs \$6 per square foot. Find the dimensions of the cheapest such container.

(10) (a) For a curve given by $y = 5e^x - 2\ln(x + 1)$, compute the equation of the tangent line at the point of the graph where $x = 0$.

(b) Simplify the following expression for $f(x)$ and use the result to compute $f'(x)$:

$$f(x) = \ln(\frac{xe^{5x}}{\sqrt{x+1}}).$$