**MATHEMATICS 205** **FINAL EXAM** **spring, 2013**

NAME ____________________________

SECTION______________

INSTRUCTOR____________________________

(1) Turn off all cell phones and put them and all notes away and out of sight.

(2) NO CALCULATORS. NO scrap paper. Use the paper provided.

(3) Leave all numerical answers in exact form. Simplify when reasonable but leave the answers in terms of $\pi$, $\sqrt{}$, $e$, ln and fractions.

(4) SHOW YOUR WORK. Points will be deducted for solutions which are not supported by your written work.

Part I Answer all questions in this part (60 POINTS).

(1) Find $\frac{dy}{dx}$ and simplify when reasonable. (5 points each):

(a) $y = \dfrac{\ln(x)}{x+1}$ (b) $y = (\sqrt{x} + x^2)^5$

(c) $y = xe^{2x} + \ln(2x+5)$. (d) $y = e^{5\ln(x)}$

(2) Compute each of the following integrals. (5 points each):

(a) $\displaystyle\int (2x+1)(x^2+1)^2\,dx$ (b) $\displaystyle\int \frac{x^2+3x+1}{x}\,dx$

(c) $\displaystyle\int (2x+1)(x^2+x+1)^5\,dx$ (d) $\displaystyle\int_1^e \frac{\ln(x^2)}{2x}\,dx$

(3) For the curve given by the equation $y = x^3 + \frac{3}{2}x^2 - 6x$

(a)(8 points) Sketch the graph, labeling the relative maxima (MAX), relative minima (MIN) and points of inflection (POI). Describe where the curve is increasing and where decreasing. Describe where the curve is concave up and where concave down.

(b) (8 points) Determine that (absolute) maximum and minimum values for $f(x) = x^3 + \frac{3}{2}x^2 - 6x$ on the closed interval $[-1, 2]$.

(4) (4 points) Let $P(t)$ be the population of a colony of fruit flies, where the time t is measured in days. The initial size of the colony was 50. The population doubles in 4 days. Assume exponential growth. What is the size of the population after 12 days?

PartII Answer all sections of four (4) questions out of the six (6) questions in this part (10 points each).

(5) A function $f$ satisfies the conditions: $f(3) = 5, f(7) = 0, f'(3) = f'(7) = 0$, and $f''(x) < 0$ when $x < 4$ while $f''(x) > 0$ when $x > 4$.

(a) Where is the derivative $f'(x)$ negative?

(b) Sketch a graph for $y = f(x)$ which is consistent with the given information.

(6) A spherical snowball with radius $r$ and volume $V = \frac{4}{3}\pi r^3$ begins to melt at 9 am. At 10 am the volume is $36\pi$ cubic inches and the volume is decreasing at a rate of $\frac{1}{10}$ cubic inches per minute. What is the rate of change of the radius at that time?

(7) (a) Compute the limits:

$$(i) Lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - x - 2}, \qquad (ii) Lim_{x \to 0} \frac{x^2 + 2}{x^2 - 1}.$$

(b) Compute the area of the bounded region between the curves $y = 4 - x^2$ and $x = 3y - 2$.

(8) (a) Assume that the function $f$ is differentiable and satisfies $f(50) = 201$ and $f'(50) = 3$. Use linear approximation to estimate $f(52)$.

(b) Use the definition of the derivative to compute the derivative of the function given by $f(x) = \sqrt{x+1}$ and show that you answer agrees with the one given by the differentiation rules.

(9) If $1200cm^2$ of material is available to make a box with a square bottom and an open top, find the dimensions of the box with the largest possible volume. Notice that all the material is used to make the sides and the bottom. None is cut out and thrown away.

(10) (a) For a curve given by $y = 3x^2 - 2x$, compute the equation of the tangent line at the point $(1, 1)$.

(b) A newspaper reported that the unemployment rate was getting worse more slowly. If $U(t)$ is the unemployment rate at time $t$ what does this say about the signs of $U'$ and $U$" at the time of the report?