MATH 205, Final Fall 2013

1. Turn off all cell phones and put them out of sight.
2. NO CALCULATORS. No scrap paper (use sheets provided).
3.Leave all numbers in exact form. Simplify answers when possible, but leave them in terms of π, ln, e and fractions.

SHOW ALL WORK

PART 1: ANSWER ALL QUESTIONS (60 points)

1. (20 Points) Find the derivative $\frac{dy}{dx}$ and simplify your answer:

(a) $y = \frac{e^x}{x^4}$ **(b)** $y = \sqrt{x^4 + 1} + ln(x^2)$

(c) $y = \frac{2x^2}{(x^2+1)^3}$ **(d)** $y = x^{\frac{3}{2}} - \frac{9}{\sqrt{x}}$

2. (6 points) Simplify the following:

(a) $e^{2\,ln(4x)}$ **(b)** $ln(e^4\,e^x)$

3. (20 Points) Compute each of the following integrals and simplify your answer.

(a) $\int_0^1 (x^2 + e^x)\,dx$ **(b)** $\int \frac{x+4}{(x^2+8x)^2}\,dx$

(c) $\int_1^2 \frac{1+x^2}{x^3}\,dx$ **(d)** $\int_4^8 \frac{1}{x-3}\,dx$

4. (5 points) A car enters the off-ramp of a highway at time $t = 0$. After t seconds its position is

$s(t) = 84t - t^3$ **feet when** $0 \leq t \leq 5$.

(a) How fast is the car going when it enters the off-ramp?
(b) Show that it really is slowing down when it enters the off-ramp.

5. (9 points) The human intestine contains Escherichia coli bacteria. In the laboratory this bacteria grows exponentially with a relative growth of $k = 0.41$ per hour. Assume that $1,000$ bacteria cells are present at time $t = 0$.

(a) Find a formula for the number $P(t)$ of bacteria at time t.
(b) How many bacteria cells will there be 5 hours from now?
(c) When will there be 10,000 cells?

PART 2: ANSWER 4 OF THE FOLLOWING 6 QUESTIONS. EACH QUESTION IS WORTH 10 POINTS. IF YOU ANSWER MORE THAN 4, CROSS OUT WORK NOT TO BE GRADED.

6. Find all the critical numbers of the function

$$f(x) = x^3 - 9x^2 + 24x - 10 \ , \qquad -\infty \le x \le \infty$$

then find all local maxima and all local minima. On what intervals is f increasing? On what intervals is f decreasing? Find all inflection points and sketch the graph.

7. The dimensions of a rectangle are changing so its width is increasing at the rate of 3 inches per second and its height is decreasing at 7 inches per second. Find how fast the area is changing when the the width is 12 inches and the area is 96 square inches.

8. (a) Sketch the curve $y = \frac{1}{x^2}$.
(b) Using a Riemann sum with 3 equal subintervals and left endpoints, estimate the value of

$$\int_1^4 \frac{1}{x^2} dx.$$

Leave your answer as a fraction. (c) Is this an over-estimate or an under-estimate?

9. (a) State the limit definition of the derivative $f'(x)$.
(b) Using that definition, compute $f'(x)$ for $f(x) = 1 - \frac{3}{x}$.
(c) What is the equation of the tangent line to $f(x) = 1 - \frac{3}{x}$ at the point $(-1, 4)$?

10. The concentration of a drug in a patient's bloodstream t hours after it is taken is given by

$$C(t) = \frac{.016t}{(t+2)^2} \ mg/cm^3 \ .$$

Find the maximum concentration and the time at which it occurs.