MATHEMATICS 205 **FINAL EXAM** **Fall, 2011**

NAME ____________________________

SECTION______________

INSTRUCTOR____________________________

(1) Turn off all cell phones and put them and all notes away and out of sight.

(2) NO CALCULATORS. NO scrap paper. Use the paper provided.

(3) Leave all numerical answers in exact form. Simplify when reasonable but leave the answers in terms of π, $\sqrt{}$, e, ln and fractions.

(4) SHOW YOUR WORK. Points will be deducted for solutions which are not supported by your written work.

Part I Answer all questions in this part (60 POINTS).

(1) Find $\frac{dy}{dx}$ and simplify when reasonable. (5 points each):

$$\text{(a)} \quad y = \frac{e^x}{x+1} \qquad \text{(b)} \quad y = xe^{(x^2+1)}$$

$$\text{(c)} \quad y = e^{2x}\ln(2x+5).$$

(2) Compute each of the following integrals. (5 points each):

$$\text{(a)} \quad \int (2x+1)(x^2+1)\, dx \qquad \text{(b)} \quad \int \frac{x^2+3}{\sqrt{x}}\, dx$$

$$\text{(c)} \quad \int (2x+1)(x^2+x+1)^5\, dx \qquad \text{(d)} \quad \int_1^e \frac{\sqrt{\ln x}}{2x}\, dx$$

$$\text{(e)} \quad \int e^x(e^x - e^{-x})\, dx.$$

(3) Simplify the following expression for $f(x)$ and use the result to compute $f'(x)$ (8 points):

$$f(x) \quad = \quad \ln(\frac{(x^2+1)e^x}{\sqrt{x}}).$$

(4) For the curve which is the graph of the equation $y = x^4 \; - \; 4x^3$

(a) Describe where the curve is increasing and where decreasing. Describe where the curve is concave up and where concave down. (6 points)

(b) Sketch the graph, labeling the relative maxima (MAX), relative minima (MIN), points of inflection (POI) and intercepts. (6 points)

PartII Answer all sections of four (4) questions out of the six (6) questions in this part (10 points each).

(5) (a) Find the coordinates of the point on the graph of the equation $y = 3x^2 + 5x - 7$ at which the curve is parallel to the line with equation $y = 17x - 14$ and compute the equation of the tangent line to the curve at that point.

(b) Compute the area of the bounded region which is bounded by the curves $y = x^2 + 2$ and $y = x + 2$.

(6) (a) For a function given by $f(x)$ write down the limit definition of the derivative $f'(x)$.

(b) Using the limit definition, compute the derivative of $f(x) = \frac{1}{2-x}$.

(c) Check the answer in (b) by computing the derivative by applying the rules for differentiation.

(7) A 10 foot long ladder leans against a vertical wall. The foot of the ladder is pulled away from the wall at the rate of 2 feet per second.

(a) Draw a diagram with x labeling the distance from the foot of the ladder to the wall and with y labeling the height of the point at which the ladder touches the wall. Write down an equation which (correctly) relates x and y. (b) Compute the rate that the top of the ladder is moving down the wall when the foot of the ladder is 8 feet from the wall.

(8) A function f satisfies the conditions: $f(3) = 5, f(7) = 0$, $f'(3) = f'(7) = 0$, and $f''(x) < 0$ when $x < 4$ while $f''(x) > 0$ when $x > 4$.

(a) Where is the derivative $f'(x)$ negative?

(b) Sketch a graph for $y = f(x)$ which is consistent with the given information.

(9) A farmer wishes to build an E shaped fence along a straight river to create two identical rectangular pastures with total area 1000 square feet. The price of the fencing parallel to the river is \$ 6 per foot and the price of the other three sections is \$ 5 per foot. Find the dimensions of the field which will minimize the total cost of the fencing. [[[Note please include a diagram with x labeling the side parallel to the river and y labeling one of the other sides]]]

(10) The size of a certain insect population is given by $P(t) = 300e^{.01t}$ where t is measured in days.

(a) What was the initial size of the population and what will the size be after 100 days?

(b) Give a differential equation satisfied by $P(t)$.

(c) At what time will the population double?