Example 1: Sketch the graph of $y = \sin(x)$ for $0 \leq x \leq 2\pi$.

- Plot the five blue points: $\sin(x) = 0$ at $x = 0, \pi, 2\pi$; $\sin\left(\frac{\pi}{2}\right) = 1$; $\sin\left(\frac{3\pi}{2}\right) = -1$
- Start at (0,0)
Basic sine graphs

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- Start at (0,0)
- Go up to point \((\pi/2, 1)\), a local maximum because it is at the top of a hill.
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Basic sine graphs

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Basic sine graphs

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- Back up to \( (2\pi, 0) \).
- Make sure your graph has no sharp corners!

The graph at the left gets half credit.
Example 2: Draw the graph of \( y = 3\sin(x) \) for \( 0 \leq x \leq 2\pi \).

The only change from the graph of \( y = \sin(x) \) is that every y-coordinate gets multiplied by 3.

- Plot the five blue points: \( 3\sin(x) = 0 \) at \( x = 0, \pi, 2\pi; \) \( \sin\left(\frac{\pi}{2}\right) = 3; \) \( \sin\left(\frac{3\pi}{2}\right) = -3 \)
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Example 2: Draw the graph of $y = 3 \sin(x)$ for $0 \leq x \leq 2\pi$.

The only change from the graph of $y = \sin(x)$ is that every $y$-coordinate gets multiplied by 3.

- Plot the five blue points: $3 \sin(x) = 0$ at $x = 0, \pi, 2\pi$; $\sin(\pi / 2) = 3$; $\sin(3\pi / 2) = -3$
- Start at (0,0)
- Go up to point $(\pi/2,3)$, a local maximum because it is at the top of a hill.
Example 2: Draw the graph of $y = 3\sin(x)$ for $0 \leq x \leq 2\pi$.

The only change from the graph of $y = \sin(x)$ is that every y-coordinate gets multiplied by 3.

- Plot the five blue points: $3\sin(x) = 0$ at $x = 0, \pi, 2\pi$; \( \sin\left(\frac{\pi}{2}\right) = 3; \quad \sin\left(\frac{3\pi}{2}\right) = -3 \)
- Start at $(0,0)$
- Go up to point $(\pi/2,3)$, a local maximum because it is at the top of a hill.
- Down to $(\pi, 0)$
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The only change from the graph of \( y = \sin(x) \) is that every y-coordinate gets multiplied by 3.

- Plot the five blue points: \( 3 \sin(x) = 0 \) at \( x = 0, \pi, 2\pi; \ \sin(\frac{\pi}{2}) = 3; \ \sin(\frac{3\pi}{2}) = -3 \)
- Start at (0,0)
- Go up to point \( (\pi/2,3) \), a local maximum because it is at the top of a hill.
- Down to \( (\pi,0) \)
- Down to point \( (3\pi/2,-3) \), a local minimum because it is at the bottom of a valley.
Example 2: Draw the graph of $y = 3\sin(x)$ for $0 \leq x \leq 2\pi$.
The only change from the graph of $y = \sin(x)$ is that every $y$-coordinate gets multiplied by 3.
- Plot the five blue points: $3\sin(x) = 0$ at $x = 0, \pi, 2\pi$; $\sin(\frac{\pi}{2}) = 3$; $\sin(\frac{3\pi}{2}) = -3$
- Start at (0,0)
- Go up to point ($\pi$/2,3), a local maximum because it is at the top of a hill.
- Down to ($\pi$,0)
- Down to point ($3\pi$/2 , -3), a local minimum because it is at the bottom of a valley.
- Back up to (2\pi, 0).
Example 2: Draw the graph of \( y = 3 \sin(x) \) for \( 0 \leq x \leq 2\pi \).

The only change from the graph of \( y = \sin(x) \) is that every y-coordinate gets multiplied by 3.

- Plot the five blue points: \( 3 \sin(x) = 0 \) at \( x = 0, \pi, 2\pi; \sin(\pi/2) = 3; \sin(3\pi/2) = -3 \)
- Start at (0,0)
- Go up to point \((\pi/2, 3)\), a local maximum because it is at the top of a hill.
- Down to \((\pi, 0)\)
- Down to point \((3\pi/2, -3)\), a local minimum because it is at the bottom of a valley.
- Back up to \((2\pi, 0)\).
- Make sure your graph has no sharp corners!
Example 3: Draw the graph of $y = -\sin(x)$ for $0 \leq x \leq 2\pi$.

This graph is the reflection through the x-axis of the red graph of $y = \sin(x)$.

- Plot the five blue points: $-\sin(x) = 0$ at $x = 0, \pi, 2\pi$; $-\sin(\frac{\pi}{2}) = -1$; $-\sin(\frac{3\pi}{2}) = 1$
- Start at (0,0)
Example 3: Draw the graph of $y = -\sin(x)$ for $0 \leq x \leq 2\pi$.
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- Plot the five blue points: $-\sin(x) = 0$ at $x = 0, \pi, 2\pi$; $-\sin(\frac{\pi}{2}) = -1$; $-\sin(\frac{3\pi}{2}) = 1$
- Start at (0,0)
- Go down to point $(\pi/2, -1)$, a local minimum because it is at the bottom of a valley.
Example 3: Draw the graph of \( y = -\sin(x) \) for \( 0 \leq x \leq 2\pi \).
This graph is the reflection through the x-axis of the red graph of \( y = \sin(x) \).

- Plot the five blue points: \( -\sin(x) = 0 \) at \( x = 0, \pi, 2\pi \); \( -\sin(\frac{\pi}{2}) = -1 \); \( -\sin(\frac{3\pi}{2}) = 1 \)
- Start at (0,0)
- Go down to point \((\pi/2, -1)\), a local minimum because it is at the bottom of a valley.
- Up to \((\pi, 0)\)
Basic sine graphs 3

Example 3: Draw the graph of $y = -\sin(x)$ for $0 \leq x \leq 2\pi$.
This graph is the reflection through the x-axis of the red graph of $y = \sin(x)$.

- Plot the five blue points: $-\sin(x) = 0$ at $x = 0, \pi, 2\pi$;  $-\sin\left(\frac{\pi}{2}\right) = -1$;  $-\sin\left(\frac{3\pi}{2}\right) = 1$
- Start at (0,0)
- Go down to point $(\pi/2,-1)$, a local minimum because it is at the bottom of a valley.
- Up to $(\pi,0)$
- Up to point $(3\pi/2 , 1)$, a local maximum because it is at the top of a hill.
**Example 3:** Draw the graph of $y = -\sin(x)$ for $0 \leq x \leq 2\pi$.

This graph is the reflection through the x-axis of the red graph of $y = \sin(x)$.

- Plot the five blue points: $-\sin(x) = 0$ at $x = 0, \pi, 2\pi$; $-\sin\left(\frac{\pi}{2}\right) = -1$; $-\sin\left(\frac{3\pi}{2}\right) = 1$
- Start at $(0,0)$
- Go down to point $(\pi/2, -1)$, a local minimum because it is at the bottom of a valley.
- Up to $(\pi, 0)$
- Up to point $(3\pi/2, 1)$, a local maximum because it is at the top of a hill.
- Back down to $(2\pi, 0)$
Memorize these four basic sine and cosine graphs.

The graph above is one wave of $y = \sin(x)$.

Reflect the above graph through the x-axis to get the following graph of $y = -\sin(x)$.

The graph above is one wave of $y = \cos(x)$.

Reflect the above graph through the x-axis to get the following graph of $y = -\cos(x)$. 
Example 4: For $0 \leq x \leq 2\pi$, draw the graphs of $y = \sin(x)$ and $y = 3\sin(x)$ on the same grid. Each graph consists of a hill and a valley, which together form one wave of the sine graph.
Transforming equations and graphs

Example 4: For $0 \leq x \leq 2\pi$, draw the graphs of $y = \sin(x)$ and $y = 3\sin(x)$ on the same grid. Each graph consists of a hill and a valley, which together form one wave of the sine graph.

Solution: The equation $y = 3\sin(x)$ is obtained by multiplying the RHS of $y = \sin(x)$ by 3. We multiply each y-coordinate on the red graph by 3 to obtain the blue graph.
**Example 4:** For $0 \leq x \leq 2\pi$, draw the graphs of $y = \sin(x)$ and $y = 3\sin(x)$ on the same grid. Each graph consists of a hill and a valley, which together form one wave of the sine graph.

**Solution:** The equation $y = 3\sin(x)$ is obtained by multiplying the RHS of $y = \sin(x)$ by 3. We multiply each y-coordinate on the red graph by 3 to obtain the blue graph.

**Example 5:** Draw the graph of $y = \sin(x)$ with $0 \leq x \leq 2\pi$. On the same grid, draw one wave of the graph of $y = \sin(2x)$. 
Transforming equations and graphs

Example 4: For $0 \leq x \leq 2\pi$, draw the graphs of $y = \sin(x)$ and $y = 3\sin(x)$ on the same grid. Each graph consists of a hill and a valley, which together form one wave of the sine graph.

Solution: The equation $y = 3\sin(x)$ is obtained by multiplying the RHS of $y = \sin(x)$ by 3. We multiply each y-coordinate on the red graph by 3 to obtain the blue graph.

Example 5: Draw the graph of $y = \sin(x)$ with $0 \leq x \leq 2\pi$. On the same grid, draw one wave of the graph of $y = \sin(2x)$.

Solution: If we replace $x$ in the equation $y = \sin(x)$ by $2x$, we get new equation $y = \sin(2x)$. The effect on the graph is to shrink it by a factor of 2 in the $x$-direction, toward the $y$-axis. The resulting blue graph still shows a complete wave (hill + valley) of the sine curve $y = \sin(2x)$. However, the domain has shrunk to $0 \leq x \leq \pi$. 
Example 6: Starting with one wave $y = \sin(x)$, draw one wave of the graph of $y = 3\sin(2x)$.
Example 6: Starting with one wave \( y = \sin(x) \), draw one wave of the graph of \( y = 3 \sin(2x) \).

Solution: We need to combine the two equation changes studied on the last slide.
- Multiply the RHS of the equation \( y = \sin(x) \) by 3 to get the new equation \( y = 3 \sin(x) \). To sketch the graph, multiply each y-coordinate on the red graph by 3 to obtain the blue graph.
- To go from the equation \( y = 3 \sin(x) \) to the desired equation \( y = 3 \sin(2x) \), replace \( x \) in the equation by \( 2x \). Thus we shrink the graph of \( y = 3 \sin(x) \) by a factor of 2 in the x-direction to obtain the black graph at the left.
- The original graph of \( y = \sin(x) \) has been stretched by a factor of 3 in the y-direction AND has been shrunk by a factor of 2 in the x-direction. The resulting black graph shows a complete wave (hill + valley) of the sine curve \( y = 3 \sin(2x) \). However, the domain has shrunk to \( 0 \leq x \leq \pi \).
Example 6: Starting with one wave $y = \sin(x)$, draw one wave of the graph of $y = 3 \sin(2x)$.

Solution: We need to combine the two equation changes studied on the last slide.

- Multiply the RHS of the equation $y = \sin(x)$ by 3 to get the new equation $y = 3 \sin(x)$. To sketch the graph, multiply each y-coordinate on the red graph by 3 to obtain the blue graph.

- To go from the equation $y = 3 \sin(x)$ to the desired equation $y = 3 \sin(2x)$, replace $x$ in the equation by $2x$. Thus we shrink the graph of $y = 3 \sin(x)$ by a factor of 2 in the x-direction to obtain the black graph at the left.

- The original graph of $y = \sin(x)$ has been stretched by a factor of 3 in the y-direction AND has been shrunk by a factor of 2 in the x-direction. The resulting black graph shows a complete wave (hill + valley) of the sine curve $y = 3 \sin(2x)$. However, the domain has shrunk to $0 \leq x \leq \pi$. 
The last slide shows a "history" of how to obtain the graph of $y = 3 \sin(2x)$ from the basic graph of $y = \sin(x)$. If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph off $y = 3 \sin(2x)$. 
The last slide shows a "history" of how to obtain the graph of $y = 3 \sin(2x)$ from the basic graph of $y = \sin(x)$. If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph off $y = 3 \sin(2x)$.

- Start by drawing the above 4 by 2 box grid, which will accommodate every cosine or sine wave.
The last slide shows a "history" of how to obtain the graph of \( y = 3 \sin(2x) \) from the basic graph of \( y = \sin(x) \). If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph of \( y = 3 \sin(2x) \).

- Start by drawing the above 4 by 2 box grid, which will accommodate every cosine or sine wave.
- To draw one wave of \( y = 3 \sin(2x) \), the angle \( 2x \) should go from 0 to \( 2\pi \).
- Thus \( x \) goes from 0 to \( \pi \).
- The width of the wave is \( \pi \), and so each section of the graph has width \( \pi/4 \).
- Start by labeling the x-axis with labels \( 0, \pi/4, \pi/2, 3\pi/4, \pi \).
The last slide shows a "history" of how to obtain the graph of $y = 3 \sin(2x)$ from the basic graph of $y = \sin(x)$.

If we just want the final result, there's an easier way.

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- To draw one wave of $y = 3 \sin(2x)$, the angle $2x$ should go from 0 to $2\pi$.
  Thus $x$ goes from 0 to $\pi$.

The width of the wave is $\pi$, and so each section of the graph has width $\pi/4$.

Start by labeling the x-axis with labels $0, \pi/4, \pi/2, 3\pi/4$.

- Next, note that the sine of any angle is between -1 and 1. If we are changing from $y = \sin(2x)$ to $3 \sin(2x)$, every y-coordinate gets multiplied by 3 and the graph wave of $y = 3 \sin(2x)$ will run from $y = -3$ to $y = 3$.

Label the y-axis with scale numbers -3, 0, 3.
The last slide shows a "history" of how to obtain the graph of \( y = 3 \sin(2x) \) from the basic graph of \( y = \sin(x) \). If we just want the final result, there's an easier way.

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- To draw one wave of \( y = 3 \sin(2x) \), the angle \( 2x \) should go from 0 to \( 2\pi \). Thus \( x \) goes from 0 to \( \pi \).
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- Start by labeling the x-axis with labels \( 0, \pi/4, \pi/2, 3\pi/4 \).
- Next, note that the sine of any angle is between -1 and 1. If we are changing from \( y = \sin(2x) \) to \( 3 \sin(2x) \), every y-coordinate gets multiplied by 3 and the graph wave of \( y = 3 \sin(2x) \) will run from \( y = -3 \) to \( y = 3 \).
- Label the y-axis with scale numbers -3, 0, 3.
- Now sketch the sine wave section by section, starting with the hill, so that it fills the box.
The last slide shows a "history" of how to obtain the graph of \( y = 3 \sin(2x) \) from the basic graph of \( y = \sin(x) \). If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph off \( y = 3 \sin(2x) \).

- Start by drawing the above 4 by 2 box grid, which will accommodate every cosine or sine wave.
- To draw one wave of \( y = 3 \sin(2x) \), the angle \( 2x \) should go from 0 to \( 2\pi \). Thus \( x \) goes from 0 to \( \pi \).
- The width of the wave is \( \pi \), and so each section of the graph has width \( \pi/4 \).
- Start by labeling the x-axis with labels 0, \( \pi/4 \), \( \pi/2 \), \( 3\pi/4 \).
- Next, note that the sine of *any* angle is between -1 and 1. If we are changing from \( y = \sin(2x) \) to \( 3 \sin(2x) \), every y-coordinate gets multiplied by 3 and the graph wave of \( y = 3 \sin(2x) \) will run from \( y = -3 \) to \( y = 3 \).
- Label the y-axis with scale numbers -3, 0, 3.
- Now sketch the sine wave section by section, starting with the hill, so that it fills the box.
The last slide shows a "history" of how to obtain the graph of $y = 3 \sin(2x)$ from the basic graph of $y = \sin(x)$. If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph off $y = 3 \sin(2x)$.

- Start by drawing the above 4 by 2 box grid, which will accommodate every cosine or sine wave.
- To draw one wave of $y = 3 \sin(2x)$, the angle $2x$ should go from 0 to $2\pi$. Thus $x$ goes from 0 to $\pi$.
  
  The width of the wave is $\pi$, and so each section of the graph has width $\pi/4$.
  Start by labeling the x-axis with labels $0, \pi/4, \pi/2, 3\pi/4$.
- Next, note that the sine of *any* angle is between -1 and 1. If we are changing from $y = \sin(2x)$ to $3 \sin(2x)$, every y-coordinate gets multiplied by 3 and the graph wave of $y = 3 \sin(2x)$ will run from $y = -3$ to $y = 3$.
  Label the y-axis with scale numbers -3, 0, 3.
- Now sketch the sine wave section by section, starting with the hill, so that it fills the box.
Basic sine and cosine graphs

The last slide shows a "history" of how to obtain the graph of \( y = 3 \sin(2x) \) from the basic graph of \( y = \sin(x) \). If we just want the final result, there's an easier way.

**Example 7:** Sketch one wave of the graph of \( y = 3 \sin(2x) \).

- Start by drawing the above 4 by 2 box grid, which will accommodate every cosine or sine wave.
- To draw one wave of \( y = 3 \sin(2x) \), the angle \( 2x \) should go from 0 to \( 2\pi \). Thus \( x \) goes from 0 to \( \pi \).
- The width of the wave is \( \pi \), and so each section of the graph has width \( \pi/4 \).
- Start by labeling the x-axis with labels 0, \( \pi/4 \), \( \pi/2 \), 3\( \pi/4 \).
- Next, note that the sine of any angle is between -1 and 1. If we are changing from \( y = \sin(2x) \) to \( 3 \sin(2x) \), every y-coordinate gets multiplied by 3 and the graph wave of \( y = 3 \sin(2x) \) will run from \( y = -3 \) to \( y = 3 \).
- Label the y-axis with scale numbers -3, 0, 3.
- Now sketch the sine wave section by section, starting with the hill, so that it fills the box.
Example 8: Graph \( y = 3 \sin(2x + \pi/3) \) by transforming the graph of \( y = \sin(x) \).

Solution: First rewrite the equation as \( y = 3 \sin(2(x + \pi/6)) \).

- Start with the graph \( y = \sin(x); 0 \leq x \leq 2\pi \).
Graphing $y = 3 \sin(2x + \pi/3)$

**Example 8:** Graph $y = 3 \sin(2x + \pi/3)$ by transforming the graph of $y = \sin(x)$.

**Solution:** First rewrite the equation as $y = 3 \sin(2(x + \pi/6))$.

- Start with the graph $y = \sin(x); 0 \leq x \leq 2\pi$.
- Replace $x$ in $y = \sin(x)$ by $2x$. This shrinks its graph horizontally toward the $y$-axis by a factor of 2 to yield one wave $y = \sin(2x); 0 \leq x \leq \pi$. 

![Graph of y = 3 sin(2x + pi/3)]
Example 8: Graph $y = 3\sin(2x + \pi/3)$ by transforming the graph of $y = \sin(x)$.

Solution: First rewrite the equation as $y = 3\sin(2(x + \pi/6))$.

- Start with the graph
  
  $y = \sin(x); 0 \leq x \leq 2\pi$.

- Replace $x$ in $y = \sin(x)$ by $2x$. This shrinks its graph horizontally toward the $y$-axis by a factor of 2 to yield one wave
  
  $y = \sin(2x); 0 \leq x \leq \pi$.

- Replace $x$ in $y = \sin(2x)$ by $x + \pi/6$. This shifts its graph left $\pi/6$ to yield one wave of
  
  $y = \sin(2(x + \pi/6)); -\pi/6 \leq x \leq 5\pi/6$.

Vertical lines are $\pi/6$ radians apart.
Graphing $y = 3\sin(2x + \pi/3)$

**Example 8:** Graph $y = 3\sin(2x + \pi/3)$ by transforming the graph of $y = \sin(x)$.

**Solution:** First rewrite the equation as $y = 3\sin(2(x + \pi/6)).$

1. Start with the graph $y = \sin(x); 0 \leq x \leq 2\pi$.
2. Replace $x$ in $y = \sin(x)$ by $2x$. This shrinks its graph horizontally toward the $y$-axis by a factor of 2 to yield one wave $y = \sin(2x); 0 \leq x \leq \pi$.
3. Replace $x$ in $y = \sin(2x)$ by $x + \pi/6$. This shifts its graph left $\pi/6$ to yield one wave of $y = \sin(2(x + \pi/6)); -\pi/6 \leq x \leq 5\pi/6$ Vertical lines are $\pi/6$ radians apart.
4. Multiply RHS of $y = \sin(2x + \pi/3)$ by 3. This stretches the graph vertically away from the $x$-axis by a factor of 2 to yield one wave of $y = 3\sin(2x + \pi/3); -\pi/6 \leq x \leq 5\pi/6$. 
Graphing \( y = 3 \sin(2x + \pi/3) \) without intermediate transformations.

On the previous slide, we started with one wave of the graph of \( y = \sin(x) \) with domain \( 0 \leq x \leq 2\pi \). We transformed the equation and its graph step by step to obtain one wave of the graph \( y = 3 \sin(2x + \pi/3) \), with domain \( -\pi/6 \leq x \leq 5\pi/6 \). This was very complicated!

Here we show how to draw the final graph directly.

**Example 9:** Graph one wave of \( y = 3 \sin(2x + \pi/3) \) directly. Label the maximum and minimum points with their coordinates.

**Solution:** The crucial idea is that one wave (hill and valley) of a sine curve is obtained by letting the angle go from 0 to \( 2\pi \). For our function \( y = \sin(2x + \pi/3) \), the angle is \( 2x + \pi/3 \).

- Angle from 0 to \( 2\pi \): \( 0 \leq 2x + \pi/3 \leq 2\pi \)
- Subtract \( \pi/3 \): \( -\pi/3 \leq 2x \leq 5\pi/3 \)
- Divide by 2: \( -\pi/6 \leq x \leq 5\pi/6 \) This is the domain of one wave of the graph.

Therefore the domain of \( y = 3 \sin(2x + \pi/3) \) will be the closed interval \([ -\pi/6, 5\pi/6 ] \).

Figuring out the \( y \)-values for one wave is easier. Since \( y = \sin(2x + \pi/3) \) can take values between -1 and 1, multiplying the RHS of by 3 multiplies all \( y \)-values by 3. Therefore the range of \( y = 3 \sin(2x + \pi/3) \), is \( -3 \leq y \leq 3 \).

**Conclusion:** One wave of the graph of \( y = 3 \sin(2x + \pi/3) \) should be drawn on a grid with \( -\pi/6 \leq x \leq 5\pi/6 \) and \( -3 \leq y \leq 3 \). We will do this on the next slide.
Graphing $y = 3 \sin(2x + \pi/3)$ without intermediate transformations.

- Draw the grid.

...
Graphing $y = 3 \sin(2x + \pi/3)$ without intermediate transformations.

- Draw the grid.
- Draw the sine wave.
Graphing $y = 3\sin(2x + \pi/3)$ without intermediate transformations.

- Draw the grid.
- Draw the sine wave.
- On the previous slide, we showed that the domain is $\frac{-\pi}{6} \leq x \leq \frac{5\pi}{6}$.
Graphing $y = 3 \sin(2x + \pi/3)$ without intermediate transformations.

• Draw the grid.
• Draw the sine wave.
• On the previous slide, we showed that the domain is $-\pi/6 \leq x \leq 5\pi/6$. This interval’s length (the width of the grid) is right endpoint minus left endpoint = $5\pi/6 - (-\pi/6) = \pi$. 

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Stanley Ocken  
M19500 Precalculus Chapter 5.3: Trigonometric graphs
Graphing $y = 3 \sin(2x + \pi/3)$ without intermediate transformations.

- Draw the grid.
- Draw the sine wave.
- On the previous slide, we showed that the domain is $-\pi/6 \leq x \leq 5\pi/6$. This interval’s length (the width of the grid) is right endpoint minus left endpoint $= \frac{5\pi}{6} - (-\frac{\pi}{6}) = \pi$.
  
  Each of the four equal parts has length $\pi/4$. The first x-axis label will be $-\frac{\pi}{6}$. To get the others, keep adding $\frac{\pi}{4}$.
  
  Therefore the other x-axis labels are:
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  Therefore the other x-axis labels are:
  - $-\frac{\pi}{6} + \frac{\pi}{4} = -\frac{2\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{12}$
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Therefore the other x-axis labels are:
- $-\frac{\pi}{6} + \frac{\pi}{4} = -2\frac{\pi}{12} + 3\frac{\pi}{12} = \frac{\pi}{4}$
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On the previous slide, we showed that the domain is \(-\pi/6 \leq x \leq 5\pi/6\). This interval’s length (the width of the grid) is right endpoint minus left endpoint = \(5\pi/6 - (-\pi/6) = \pi\).

Each of the four equal parts has length \(\pi/4\). The first x-axis label will be \(-\pi/6\). To get the others, keep adding \(\pi/4\).

Therefore the other x-axis labels are:

- \(-\pi/6 + \pi/4 = -2\pi/12 + 3\pi/12 = \pi/12\)
- \(\pi/12 + \pi/4 = \pi/12 + 3\pi/12 = 4\pi/12 = \pi/3\)
- \(\pi/3 + \pi/4 = 4\pi/12 + 3\pi/12 = 7\pi/12\) and finally
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- Now insert y-axis labels, going from -3 to 3.
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  - $\pi/3 + \pi/4 = 4\pi/12 + 3\pi/12 = 7\pi/12$ and finally
  - $7\pi/12 + \pi/4 = 7\pi/12 + 3\pi/12 = 10\pi/12 = 5\pi/6$
- Now insert y-axis labels, going from -3 to 3
- Label the maximum and minimum points.
The procedure on the previous page for graphing $y = 3 \sin(2x + \pi/3)$ is easier to understand if we consider the general case.

The general sine function is $y = A \sin(Bx + C)$. In our example $y = 3 \sin(2x + \pi/3)$. Therefore $A = 3$, $B = 2$, and $C = \pi/3$.

**Features of the sine function graph $y = A \sin(Bx + C)$**

- **The period** is the width of one wave $= 2\pi/|B|$.
- **The amplitude** is half the height of the wave $= |A|$.
- **The phase shift** is the wave’s leftmost x-value $= -C/B$.

**Example 10:** Find the period, amplitude, and phase shift of the sine graph $y = 3 \sin(2x + \pi/3)$.

**Solution:** Match the general function $y = A \sin(Bx + C)$ with $y = 3 \sin(2x + \pi/3)$ to get $A = 3$, $B = 2$, and $C = \pi/3$.

**Answer:**
- The period is the width of one wave $= 2\pi/|B| = 2\pi/2 = \pi$.
- The amplitude is half the height of the wave $= |A| = 3$.
- The phase shift is its left x-value $= -C/B = -\pi/3 = -\pi/6$. 
Quiz Review

Example 1: Sketch the graph of \( y = \sin(x) \) for \( 0 \leq x \leq 2\pi \).

Example 2: Draw the graph of \( y = 3 \sin(x) \) for \( 0 \leq x \leq 2\pi \).

Example 3: Draw the graph of \( y = -\sin(x) \) for \( 0 \leq x \leq 2\pi \).

Example 4: For \( 0 \leq x \leq 2\pi \), draw the graphs of \( y = \sin(x) \) and \( y = 3 \sin(x) \) on the same grid.

Example 5: For \( 0 \leq x \leq 2\pi \), draw the graphs of \( y = \sin(x) \) and \( y = \sin(2x) \) on the same grid.

Example 6: Starting with one wave of \( y = \sin(x) \), draw one wave of the graph of \( y = 3 \sin(2x) \).

Example 7: Sketch one wave of the graph of \( y = 3 \sin(2x) \) directly.

Example 8: Graph \( y = 3 \sin(2x + \pi/3) \) by transforming one wave of the graph of \( y = \sin(x) \).

Example 9: Sketch one wave of \( y = 3 \sin(2x + \pi/3) \) directly. Label the maximum and minimum points with their coordinates.

Example 10: Find the period, amplitude, and phase shift of the sine graph \( y = 3 \sin(2x + \pi/3) \).