When you solve the equation $x^2 = 9$, the answer is written as two very simple equations: $x = 3$ (or) $x = -3$ The diagram of the solution is

We will soon see that the solution of the inequality $x^2 < 9$ is written in inequality form as $-3 < x$ and $x < 3$. This means that $x$ is both larger than $-3$ and less than 3. In other words, $x$ is strictly between -3 and 3.

The answer in interval form is $x$ in $(-3, 3)$. The diagram is:

We can always express the solution as an interval or as a union (collection) of intervals. For example, we will see later that the solution of the inequality $x(x + 1)(x - 3) \geq 0$ is $[-1, 0] \cup [3, \infty)$. In other words $x(x + 1)(x - 3) \geq 0$ if either

- $x$ in $[-1, 0]$ (so $x$ is 0, $x = 1$, or $x$ is between 0 and 1) OR
- $x$ in $[3, \infty)$. (so $x$ is 3 or $x$ is to the right of 3).

The diagram of this solution consists of two non-overlapping intervals:
When you solve an equation, you rewrite it in simpler form by adding any expression to both sides, or by multiplying or dividing both sides by any nonzero number. When you solve an inequality, there is one important change:

**Permitted moves for solving inequalities**

- You may add (subtract) any expression to (from) both sides of an inequality.
- You may multiply or divide both sides of an inequality by a strictly positive (greater than 0) number.
- You may multiply or divide both sides of an inequality by a strictly negative (less than 0) number PROVIDED you switch the inequality sign.
- You must not multiply or divide both sides of an inequality by an expression in $x$ unless you know that the expression is either positive for all $x$ or negative for all $x$.

Switch the inequality sign means that you change every less than sign to a greater than sign, and vice versa. The reason for this rule is that multiplying numbers on the number line by a negative number exchanges left and right: $3 < 4$ but $-3 > -4$. 
Solving absolute value inequalities.

Recall that $|x|$, the absolute value of $x$, is the distance from $x$ to 0 on the number line.

- $|x| < 3$ says $-3 < x < 3$: the distance between $x$ and 0 is less than 3.

![Graph showing the interval $-3 < x < 3$.]

Note that $-3 < x < 3$ means $-3 < x$ AND $x < 3$.

- $|x| > 3$ says $x < -3$ or $3 < x$: the distance between $x$ and 0 is more than 3.

![Graph showing the intervals $x < -3$ and $3 < x$.]

We can substitute any expression (for example, $2x + 4$) for $x$ in the above statements.

- $|2x + 4| < 3$ says $-3 < 2x + 4 < 3$.

- $|2x + 4| > 3$ says $2x + 4 < -3$ or $3 < 2x + 4$. 

![Graph showing the intervals $-3 < 2x + 4 < 3$ and $2x + 4 < -3$.]
Example 1: Solve $|2x + 4| < 3$
Rewrite as $-3 < 2x + 4 < 3$
Subtract 4 $-7 < 2x < -1$
Divide by 2 $-7/2 < x < -1/2$. This is the answer as an inequality.
$(-7/2, -1/2)$. This is the answer in interval notation.

Example 2: Solve $|2x + 4| \geq 3$
Rewrite as $2x + 4 \leq -3$ or $3 \leq 2x + 4$
Subtract 4 $2x \leq -7$ or $-1 \leq 2x$
Divide by 2 $x \leq -7/2$ or $-1/2 \leq x$. This is the answer as an inequality.
$(-\infty, -7/2] \cup [-1/2, \infty)$. This is the answer in interval notation.
We discussed earlier how to solve polynomial equations $P(x) = Q(x)$. Now we discuss solving polynomial inequalities. For example, solving $P(x) < Q(x)$ means: find all intervals of real numbers $x$ for which $P(x)$ is less than $Q(x)$.

Four different kinds of inequalities need to be discussed:

$$P(x) > Q(x) \text{ or } P(x) < Q(x) \text{ or } P(x) \geq Q(x) \text{ or } P(x) \geq Q(x).$$

We will often given an explanation using just one inequality sign and leave it you to rewrite the explanation using the other three inequality signs.

To simplify the above four inequalities, subtract $Q(x)$ from both sides and set $p(x) = P(x) - Q(x)$, to obtain:

$$p(x) > 0 \text{ or } p(x) < 0 \text{ or } p(x) \geq 0 \text{ or } p(x) \leq 0$$
The difficulty of solving $p(x) > 0$ depends on the degree of $p(x)$. When the degree is one, we have a linear inequality. In the following we abbreviate statements such as "Add 4 to both sides" by "Add 4."

- Solve $x - 4 > 0$: Add 4 to get $x > 4$ with solution $(4, \infty)$.
- Solve $x - 4 < 0$: Add 4 to get $x < 4$ with solution $(-\infty, 4)$.
- Solve $3x - 4 \geq 0$: Add 4, then divide by 3 to get $x \geq 4/3$ with solution $[4/3, \infty)$.
- Solve $3x - 4 \leq 0$: Add 4, then divide by 3 to get $x \leq 4/3$ with solution $(-\infty, 4/3]$.

Be careful when $x$ has a negative coefficient:

- Solve $4 - x > 0$: Add $x$ to get $4 > x$, the interval $(-\infty, 4)$.
- Solve $4 - 3x < 0$: Add $3x$, then divide by 3 to get $4/3 < x$, the interval $(4/3, \infty)$.
- Solve $-5x \geq 4$: Divide by -5 to get $x \leq -4/5$, the interval $(-\infty, -4/5]$.
- Solve $-6 < -2x$: Divide by -2 to get $3 > x$, the interval $(x, 3]$.
Solving polynomial inequalities

**The test point principle**

*If a polynomial \( p(x) \) is never zero on an interval, then either*

- \( p(x) > 0 \) for all \( x \) in the interval OR
- \( p(x) < 0 \) for all \( x \) in the interval.

We use this principle to solve polynomial inequalities as follows:

**Solving polynomial inequalities**

- *Use the solutions of \( p(x) = 0 \) to split the number line into (open) intervals.*
- *Choose any convenient test point \( x \) inside each interval and find the sign of \( p \) at that test point. This is the sign of \( p(x) \) in the entire interval.*
- *Solution of \( p(x) > 0 \): all intervals in which \( p \) is +. Omit endpoints.*
- *Solution of \( p(x) \geq 0 \): all intervals in which \( p \) is +. Include endpoints.*
- *Solution of \( p(x) < 0 \): all intervals in which \( p \) is −. Omit endpoints.*
- *Solution of \( p(x) \leq 0 \): all intervals in which \( p \) is +. Include endpoints.*
Example 3: Solve $x^2 + 3 \geq 4x$

Solution: First rewrite the inequality as $p(x) = x^2 - 4x + 3 \geq 0$

Step 1. $p(x) = (x - 3)(x - 1) = 0$ at critical numbers $x = 1$ and $x = 3$.

Step 2. The test intervals are $(-\infty, 1)$, $(1, 3)$ and $(3, \infty)$.

Step 3. Choose test points $0, 2, 4$ (one in each interval).

Step 4. Calculate $p(x) = (x - 3)(x - 1)$ at each test point:

$p(0) = (-1)(-3) = 3; \quad p(2) = (-1)(1) = -1; \quad p(4) = (1)(3) = 3.$

You may use a sign chart:

<table>
<thead>
<tr>
<th></th>
<th>$(-\infty, 1)$</th>
<th>$(1, 3)$</th>
<th>$(3, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test point $x =$</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>$p(x) = (x - 3)(x - 1)$ is</td>
<td>+3</td>
<td>-1</td>
<td>+3</td>
</tr>
<tr>
<td>Sign of $p(x)$ is</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

OR a number line to record the sign of $p(x)$ on each test interval:

Test intervals: $(-\infty, 1)$, $(1, 3)$, $(3, \infty)$

$p(x)$ at test points: $p(0) = +3$, $p(2) = -1$, $p(4) = +3$

Since we are solving $p(x) \geq 0$, the answer consists of intervals on which the sign of $p(x)$ is $+$, including endpoints so that $p(x)$ can be 0.

Answer: $(-\infty, 1] \cup [3, \infty)$. 
To check your answer you should always go back to the original problem, which in this case is \( x^2 + 3 \geq 4x \), not \( p(x) = (x - 1)(x - 3) \geq 0 \). You can use WolframAlpha to check your answer by graphing \( y = x^2 + 3 \) and \( y = 4x \) to see which is larger. You will get a picture like (albeit not as beautiful as) the one at the right:

Clearly \( x^2 + 3 \geq 4x \) for \( x \leq 1 \) and \( 3 \leq x \).

In interval notation: for \( x \) in \( (-\infty, 1] \cup [3, \infty) \).

In these notes we prefer to express all inequalities with < or \( \leq \) signs. That’s why we wrote \( 3 \leq x \) rather than \( x \geq 3 \).
Example 4: Solve $x^3 > x$

Solution: First rewrite the inequality as $x^3 - x > 0$.

- Factor and solve $p(x) = x^3 - x = x(x^2 - 1) = x(x - 1)(x + 1) = 0$.
- The solutions $x = -1, 0, 1$ split the line into intervals listed below.
- Choose a test point $x$ in each interval and find the sign of $p(x)$.

Rather than computing $p(x) = x^3 - x$ directly (and working with messy fractions) we factor $p(x)$, find the sign of each factor, and then multiply signs to find the sign of $p(x)$ at the test point, all shown below. We will write $--=+$ to abbreviate "negative number times negative number = positive number."

<table>
<thead>
<tr>
<th>In interval</th>
<th>$(\infty, -1)$</th>
<th>$(-1, 0)$</th>
<th>$(0, 1)$</th>
<th>$(1, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test point $x$ is</td>
<td>$-2$</td>
<td>$-1/2$</td>
<td>$1/2$</td>
<td>$2$</td>
</tr>
<tr>
<td>Factor $x$ is</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Factor $x - 1$ is</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>Factor $x + 1$ is</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$x(x - 1)(x + 1) = p(x)$ is</td>
<td>$--=-$</td>
<td>$--++=+$</td>
<td>$-++=+$</td>
<td>$+++=+$</td>
</tr>
</tbody>
</table>
The solution of \( p(x) = x^3 - x > 0 \) consists of the intervals on which \( p(x) \) is positive, omitting endpoints.

From the last line of the chart, \( p(x) \) is positive on \((-1, 0)\) and \((1, \infty)\).

**Answer:**
Interval form: \((-1, 0) \cup (1, \infty)\).
Inequality form: \(-1 < x < 0 \) or \(1 < x\).

Instead of using the chart above, you can use the following number line picture, which displays the same information in a different format.

<table>
<thead>
<tr>
<th>Test intervals:</th>
<th>((-\infty, -1))</th>
<th>(-1)</th>
<th>((-1, 0))</th>
<th>(0)</th>
<th>((0, 1))</th>
<th>(1)</th>
<th>((1, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test points:</td>
<td>(x = -2)</td>
<td>-</td>
<td>(x = -1/2)</td>
<td>-</td>
<td>(x = 1/2)</td>
<td>-</td>
<td>(x = 2)</td>
</tr>
<tr>
<td>Sign of</td>
<td>(- - - = -)</td>
<td></td>
<td>(- - + = +)</td>
<td></td>
<td>(- + + = -)</td>
<td></td>
<td>(+ + + = +)</td>
</tr>
<tr>
<td>((x + 1)(x)(x - 1) = p(x))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 5:** Use this number line sign chart to solve \( x^3 \leq x \).
**Solution:** This inequality becomes \( p(x) = x^3 - x \leq 0 \). Since \( p(x) \) is the same as in the last problem, we use the above sign chart, but now we want all intervals on which \( p(x) \) is negative, including endpoints to allow \( p(x) = 0 \).
**Answer:** \((-\infty, -1] \cup [0, 1]\); in inequality form: \(x \leq -1\) or \(0 \leq x \leq 1\).
Example 6: Solve $x^3 \geq 2x^2 + 3x$

Solution:

- First rewrite as and solve $p(x) \geq 0$.
  
  \[
x^3 - 2x^2 + 3x \geq 0
  \]
  
  \[
x(x^2 - 2x - 3) \geq 0
  \]
  
  \[
x(x + 1)(x - 3) \geq 0
  \]

- Solve $p(x) = x(x + 1)(x - 3) = 0$ to find the critical numbers 0, −1, and 3.

- These solutions split the number line into four intervals:
  
  \((-\infty, -1), \ (-1, 0), \ (0, 3), \ \text{and} \ (3, \infty)\).

- Choose one test point in each interval, as shown on the next frame.
• Fill in the signs of each factor at each test point. Add a line at the bottom that shows the sign of \( p(x) \) at each test point, which you find by multiplying the signs of the factors above it in the chart.

<table>
<thead>
<tr>
<th>For ( x ) in the interval</th>
<th>((-\infty, -1))</th>
<th>((-1, 0))</th>
<th>((0, 3))</th>
<th>((3, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choose test point ( x = )</td>
<td>(-2)</td>
<td>(-1/2)</td>
<td>(1)</td>
<td>(4)</td>
</tr>
<tr>
<td>Factor ((x + 1)) is</td>
<td>(-)</td>
<td>(+)</td>
<td>(+)</td>
<td>(+)</td>
</tr>
<tr>
<td>Factor (x) is</td>
<td>(-)</td>
<td>(-)</td>
<td>(+)</td>
<td>(+)</td>
</tr>
<tr>
<td>Factor ((x - 3)) is</td>
<td>(-)</td>
<td>(-)</td>
<td>(-)</td>
<td>(+)</td>
</tr>
<tr>
<td>(p(x) = x(x + 1)(x - 3)) is</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
</tr>
<tr>
<td>(p(x) \geq 0) in</td>
<td>([-1, 0])</td>
<td></td>
<td></td>
<td>([3, \infty))</td>
</tr>
</tbody>
</table>

Finally, express the solution of the inequality in interval form.

\[ x(x + 1)(x - 3) \geq 0 \text{ when } p(x) \text{ is } + \text{ or } p(x) = 0. \]

\[ p(x) > 0 \text{ in } (-1, 0) \cup (3, \infty) \]

\[ p(x) = 0 \text{ for } x = 0, -1, 3. \] Combine this line and the last one.

**Answer:**

• \( p(x) \geq 0 \text{ for } x \text{ in } [-1, 0] \cup [3, \infty) \) (interval form)

• \( p(x) \geq 0 \text{ for } -1 \leq x \leq 0 \text{ or } 3 \leq x. \) (inequality form)
Solving rational inequalities

Definitions

- A **polynomial fraction** is a fraction in which numerator and denominator are polynomials.
- A **rational expression** is any expression that includes at least one fraction.
- A **rational inequality** is any inequality involving a rational expression.

A simple example of a rational inequality is \( \frac{x}{x + 1} + 2 > \frac{3}{x} \)

**WARNING:** If this were an equation \( \frac{x}{x + 1} + 2 = \frac{3}{x} \), you would begin by multiplying both sides by the LCD \( x(x + 1) \) of the fractions.

However, the problem above is an inequality, not an equation.

**DO NOT** multiply both sides of \( \frac{x}{x + 1} + 2 > \frac{3}{x} \) by the LCD \( x(x + 1) \) of the fractions.

Reason: the sign of \( x(x + 1) \) will be different for different values of \( x \). According to the inequality rules, you would then have to keep the inequality sign when \( x(x + 1) > 0 \) but switch it when \( x(x + 1) > 0 \). There is no reasonable way to do this.
To solve a rational inequality, first, subtract the right side from the left side and combine fractions. This yields an inequality in the form \( \frac{p(x)}{q(x)} > 0 \) or \( \frac{p(x)}{q(x)} < 0 \) or \( \frac{p(x)}{q(x)} \geq 0 \) or \( \frac{p(x)}{q(x)} \leq 0 \).

**Procedure for solving rational inequalities**

- **Solve both** \( p(x) = 0 \) and \( q(x) = 0 \). **The solutions split the number line into (open) test intervals.**
- **Choose any convenient test point** \( x \) inside each interval and find the sign of \( p \) at that test point. **This is the sign of** \( \frac{p(x)}{q(x)} \) **in the entire interval.**
- **Solution of** \( \frac{p(x)}{q(x)} > 0 \): **all intervals in which** \( \frac{p(x)}{q(x)} \) **is +. Omit endpoints.**
- **Solution of** \( \frac{p(x)}{q(x)} \geq 0 \): **all intervals in which** \( \frac{p(x)}{q(x)} \) **is +. Include endpoints** \( x \) **satisfying** \( p(x) = 0 \)
- **Solution of** \( \frac{p(x)}{q(x)} < 0 \): **all intervals in which** \( \frac{p(x)}{q(x)} \) **is −. Omit endpoints.**
- **Solution of** \( \frac{p(x)}{q(x)} \leq 0 \): **all intervals in which** \( \frac{p(x)}{q(x)} \) **is −. Include endpoints** \( x \) **satisfying** \( p(x) = 0 \)
Example 7: Solve \( \frac{x + 2}{x + 3} \leq \frac{x - 1}{x - 2} \)

First move everything to one side and combine fractions.

DO NOT multiply by the LCD \((x + 3)(x - 2)\).

Everything to one side: \( \frac{x + 2}{x + 3} - \frac{x - 1}{x - 2} \leq 0 \)

Combine fractions: \( \frac{(x + 2)(x - 2) - (x - 1)(x + 3)}{(x + 3)(x - 2)} \leq 0 \)

Parenthesis Rule 2!

\( \frac{x^2 - 4 - (x^2 + 2x - 3)}{(x + 3)(x - 2)} \leq 0 \)

Distribute the minus sign: \( \frac{x^2 - 4 - x^2 - 2x + 3}{(x + 3)(x - 2)} \leq 0 \)

Rewrite the numerator: \( \frac{-2x - 1}{(x + 3)(x - 2)} \leq 0 \).

Thus \( p(x) = -2x - 1 \) and \( q(x) = (x + 3)(x - 2) \).

We want \( p(x)/q(x) \) to be negative or zero.

Step 1: Set \( p(x) = 0 \) and \( q(x) = 0 \) to find the critical numbers \(-3, -\frac{1}{2}, \) and \( 2 \).

Step 2: These split the number line into test intervals\((-\infty, -3); \ (-3, -1/2) \ ; \ (-1/2, 2) \) and \( (2, \infty) \)
Step 3: Choose one test point in each interval \( x = -4, -1, 1, 4 \).

Step 4: Fill in the chart below.
The sign of \( \frac{p(x)}{q(x)} \) in each interval is the product of the signs above it.

<table>
<thead>
<tr>
<th>In interval</th>
<th>((-\infty, -3))</th>
<th>((-3, -1/2))</th>
<th>((-1/2, 2))</th>
<th>((2, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test point is ( x = )</td>
<td>-4</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>(x + 3) is</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>(-2x - 1) is</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(x - 2) is</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>(p(x)/q(x)) is</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>(p(x)/q(x) \leq 0) in</td>
<td>((-3, -1/2])</td>
<td></td>
<td></td>
<td>((2, \infty))</td>
</tr>
</tbody>
</table>

The solution of the (strict) inequality \( \frac{x+2}{x+3} - \frac{x-1}{x-2} \leq 0 \) consists of all intervals on which \( \frac{p(x)}{q(x)} = \frac{-2x-1}{(x+3)(x-2)} \) is negative or zero.

This is true in intervals \((-3, -1/2]\) and \((2, \infty)\).

We include \( x = -\frac{1}{2} \) since it is a root of \( p(x) \) and so makes \( \frac{p(x)}{q(x)} = 0 \).

We omit \( x = 2 \) or \(-3\) since these are roots of \( q(x) \) and so make \( \frac{p(x)}{q(x)} \) undefined.

Answer: \((-3, -1/2]\) \( \cup \) \((2, \infty)\).
Example 8: Solve \( \frac{x + 2}{x + 3} - \frac{x - 1}{x - 2} < 0 \).

Solution: This is the same as the previous example, except that the inequality sign has been changed from \( \leq \) (less than or equal) to \( < \) (strictly less than). Moving everything to one side gives \( \frac{p(x)}{q(x)} = \frac{-2x - 1}{(x + 3)(x - 2)} < 0 \). The only change from the previous problem is: omit \( x = -1/2 \) since \( p(-1/2) = 0 \).

Answer: \((-3, -1/2) \cup (2, \infty)\).
Example 9: Solve \( \frac{3}{x} \leq 1 + \frac{2}{x^2} \)

The problem is to solve: \( \frac{3}{x} \leq 1 + \frac{2}{x^2} \)

Everything to one side: \( \frac{3}{x} - 1 - \frac{2}{x^2} \leq 0 \)

Rewrite with LCD \( x^2 \): \( \frac{3x - x^2 - 2}{x^2} \leq 0 \)

Combine fractions: \( \frac{3x-x^2-2}{x^2} \leq 0 \)

Multiply by -1: \( \frac{x^2-3x+2}{x^2} \geq 0 \). Remember to switch \( \leq \) to \( \geq \)

Factor the numerator: \( \frac{(x-1)(x-2)}{x^2} \geq 0 \) is the rewritten inequality.

Setting \( (x - 1)(x - 2) = 0 \) and \( x^2 = 0 \) yields critical numbers 0, 1 and 2.

These split the line into test intervals \( (-\infty, 0); \ (0, 1); \ (1, 2); \) and \( (2, \infty) \).

As usual, we choose one test point in each interval. See the next slide.
The sign chart from the previous slide is:

<table>
<thead>
<tr>
<th>In interval</th>
<th>$(-\infty, 0)$</th>
<th>$(0, 1)$</th>
<th>$(1, 2)$</th>
<th>$(2, \infty)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test point is $x =$</td>
<td>$-1$</td>
<td>$1/2$</td>
<td>$3/2$</td>
<td>$3$</td>
</tr>
<tr>
<td>$(x - 1)$ is</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(x - 2)$ is</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$(x^2)$ is</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\frac{p(x)}{q(x)} = \frac{(x-1)(x-2)}{x^2}$ is</td>
<td>$+$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+$</td>
</tr>
<tr>
<td>$\frac{(x-1)(x-2)}{x^2} \geq 0$ in</td>
<td>$(-\infty, 0)$</td>
<td>$(0, 1]$</td>
<td>$[2, \infty)$</td>
<td></td>
</tr>
</tbody>
</table>

and we want to solve the rewritten inequality $\frac{(x - 1)(x - 2)}{x^2} \geq 0$.

- The sign chart says "positive" for intervals $(-\infty, 0)$ and $(0, 1)$ and $(2, \infty)$.
- Include $x = 1$ and $x = 2$ because they make $\frac{(x - 1)(x - 2)}{x^2}$ equal to 0.
- Omit $x = 0$, which makes the denominator 0 and the fraction undefined.

Answer: $(-\infty, 0) \cup (0, 1] \cup [2, \infty)$. 
To check your answer we go back to the original problem, which in this case is

\[ \frac{3}{x} \leq 1 + \frac{2}{x^2} \]

You can use WolframAlpha to check your answer by graphing \( y = \frac{3}{x} \) and \( y = 1 + \frac{2}{x^2} \) to see which is larger. You will get a picture like (albeit not as beautiful as) the one at the right:

Clearly \( \frac{3}{x} \leq 1 + \frac{2}{x^2} \) for \( x < 0 \) and \( 0 < x \leq 1 \) and \( 2 \leq x \).

In interval notation:
\( (-\infty, 0) \cup (0, 1] \cup [2, \infty) \).
At a right is a closeup of the interesting part of the previous diagram, where \( x \geq 0 \).

Clearly \( \frac{3}{x} \leq 1 + \frac{2}{x^2} \) for \( 0 < x \leq 1 \) or \( 2 \leq x \); in interval notation: for \( x \) in \((0, 1] \cup [2, \infty)\).
Example 1: Solve \(|2x + 4| < 3\).
Example 2: Solve \(|2x + 4| \geq 3\).
Example 3: Solve \(x^2 + 3 \geq 4x\).
Example 4: Solve \(x^3 > x\).
Example 5: Solve \(x^3 \leq x\).
Example 6: Solve \(x^3 \geq 2x^2 + 3x\).
Example 7: Solve \(\frac{x + 2}{x + 3} \leq \frac{x - 1}{x - 2}\).
Example 8: Solve \(\frac{x + 2}{x + 3} - \frac{x - 1}{x - 2} < 0\).
Example 9: Solve \(\frac{3}{x} \leq 1 + \frac{2}{x^2}\).