Right Triangles

Tamara Kucherenko
We define sine, cosine, tangent, cosecant, secant, cotangent as

<table>
<thead>
<tr>
<th>Trigonometric Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} )</td>
</tr>
<tr>
<td>( \csc \theta = \frac{\text{hypotenuse}}{\text{opposite}} )</td>
</tr>
</tbody>
</table>

The trigonometric ratios depend only on the angle \( \theta \) and not on the size of the triangle. The reason is that any two right triangles with angle \( \theta \) are similar and the ratios of their corresponding sides are the same.
Example 1. Find the six trigonometric ratios of the angle $\theta$.

$$\sin \theta = \frac{2}{\sqrt{13}}$$
$$\cos \theta = \frac{3}{\sqrt{13}}$$
$$\tan \theta = \frac{2}{3}$$
$$\csc \theta = \frac{\sqrt{13}}{2}$$
$$\sec \theta = \frac{\sqrt{13}}{3}$$
$$\cot \theta = \frac{3}{2}$$

Example 2. If $\sin \theta = \frac{4}{7}$, sketch a right triangle with acute angle $\theta$.

Solution. Since $\sin \theta$ is defined as the ratio of the opposite side to the hypotenuse, we sketch a triangle with a hypotenuse of length 7 and a side of length 4 opposite to $\theta$.

To find the adjacent side we use the Pythagorean Theorem:

$$\text{(adjacent)}^2 = 7^2 - 4^2 = 33$$

so the adjacent side is $\sqrt{33}$. 

\[ \theta \]

\[ \frac{4}{7} \]

\[ \sqrt{33} \]
Example 1. Find the six trigonometric ratios of the angle \( \theta \).

Solution. 
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\sin \theta = \frac{2}{\sqrt{13}}, \quad \cos \theta = \frac{3}{\sqrt{13}}, \quad \tan \theta = \frac{2}{3}, \\
\csc \theta = \frac{\sqrt{13}}{2}, \quad \sec \theta = \frac{\sqrt{13}}{3}, \quad \cot \theta = \frac{3}{2}
\]
Trigonometric Ratios

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\]
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![Right Triangle Diagram]
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Solution. $\sin \theta = \frac{2}{\sqrt{13}}$  $\cos \theta = \frac{3}{\sqrt{13}}$  $\tan \theta = \frac{2}{3}$

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**Solution.** Since \( \sin \theta \) is defined as the ratio of the opposite side to the hypotenuse, we sketch a triangle with hypotenuse of length 7 and a side of length 4 opposite to \( \theta \). To find the adjacent side we use the Pythagorean Theorem:
\[
(adjacent)^2 = 7^2 - 4^2 = 33, \quad \text{so adjacent} = \sqrt{33}.
\]
We can use the special triangles below to calculate the trigonometric ratios for angles with measures $30^\circ$, $45^\circ$, and $60^\circ$. 

- For a $45^\circ$ triangle, the sides are equal, and the ratio of the sides is $1:1:\sqrt{2}$.
- For a $30^\circ$-$60^\circ$ triangle, the sides are in the ratio $1:2:2\sqrt{3}$.

You do not need to, and should not, memorize this table. Rather, memorize the two red triangles and be able to quickly calculate all trig functions of $30^\circ$, $45^\circ$, or $60^\circ$. 
We can use the special triangles below to calculate the trigonometric ratios for angles with measures $30^\circ$, $45^\circ$, and $60^\circ$.

![Special Triangles Diagram]

<table>
<thead>
<tr>
<th>$\theta$ in degrees</th>
<th>$\theta$ in radians</th>
<th>$\sin \theta$</th>
<th>$\cos \theta$</th>
<th>$\tan \theta$</th>
<th>$\csc \theta$</th>
<th>$\sec \theta$</th>
<th>$\cot \theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$30^\circ$</td>
<td>$\frac{\pi}{6}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{\sqrt{3}}{3}$</td>
<td>$2$</td>
<td>$\frac{2\sqrt{3}}{3}$</td>
<td>$\sqrt{3}$</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>$\frac{\pi}{4}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$\frac{\sqrt{2}}{2}$</td>
<td>$1$</td>
<td>$\sqrt{2}$</td>
<td>$\sqrt{2}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>$\frac{\pi}{3}$</td>
<td>$\frac{\sqrt{3}}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\sqrt{3}$</td>
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You do not need to, and should not, memorize this table. Rather, memorize the two red triangles and be able to quickly calculate all trig functions of $30\,\text{ or } 45\,\text{ or } 60\,\text{ degrees}$.
Applications of Trigonometry of Right Triangles

To **solve a triangle** means to determine all of its parts from the information known about the triangle, that is, to determine the lengths of the three sides and the measures of the three angles.

**Example.** Solve the right triangle

![Right Triangle Diagram]

To find $AC$, we look for an equation that relates $AC$ to the lengths and angles we already know. In this case, we have $\sin \frac{\pi}{3} = \frac{AC}{AB} = \frac{AC}{5}$. Thus, $AC = 5 \sin \frac{\pi}{3} = 5 \frac{\sqrt{3}}{2}$.

Similarly, $BC = 5 \cos \frac{\pi}{3} = 5 \frac{1}{2}$. 
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Example. Solve the right triangle

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Solution. First, $\angle A = \frac{\pi}{2} - \frac{\pi}{3} = \frac{\pi}{6}$.

To find $AC'$, we look for an equation that relates $AC'$ to the lengths and angles we already know. In this case, we have $\sin \frac{\pi}{3} = \frac{AC}{AB} = \frac{AC}{5}$. Thus,

$$AC = 5 \sin \frac{\pi}{3} = \frac{5\sqrt{3}}{2}.$$

Similarly, $BC = 5 \cos \frac{\pi}{3} = \frac{5}{2}$.
```