Getting basic information from the graph of a function

A function is a collection of number pairs \((x, y)\), where \(x\) is an input and \(y = f(x)\) is the corresponding output. Here are some models that give rise to functions.

- **Manufacturing:** Input \(x\) is how many tons of bricks a factory sells. Output \(f(x)\) is net profit: the sale price of \(x\) bricks minus their cost.

- **Saving the dolphins:** Input \(t\) is the number of hours after Noon. Output \(f(t)\) is the gap between the radius of a circular oil slick spreading from an oil tanker explosion and a dolphin swimming away from the explosion, as in Section 1.6.

- **Moving particle:** Input \(t\) is the number of seconds after Noon. Output \(f(t)\) is the height above ground at time \(t\) of a ball thrown up from ground at Noon.

Each of these physical situations is modeled by a different function \(f\). However, the information we would like to know about each model can usually be obtained by asking and answering the same basic questions about the graph of \(f\).
Let’s use the moving particle model. Suppose a ball is thrown up at time $t = 0$ and released with velocity 96 feet per second. Newtonian physics says that its height above the ground (forgetting about air resistance, and only until it comes back down) is $y = h(t) = 96t - 16t^2$ feet. The ball starts at ground level $y = 0$, then goes up to a highest point, and then goes down until it hits the ground. Below is the graph of $y = h(t) = 96t - 16t^2$.

Here are five important questions:

- What’s the connection between the picture and the formula?
- Suppose you know the output $y = h(t)$. What input or inputs $t$ produced that output?
- What is the maximum value of $y = h(t)$? What is the minimum value?
- In what (time) interval of inputs is $y = h(t)$ getting larger?
- In what (time) interval is $y = h(t)$ getting smaller?
Suppose $f$ is a function whose domain contains an interval $I$.

**Definitions (these will make better sense after you look at the slides that follow).**

- $f$ is **increasing** on $I$ means: $f(a) < f(b)$ for any two points $a$ and $b$ in $I$ with $a < b$.
- $f$ is **decreasing** on $I$ means: $f(a) > f(b)$ for any two points $a$ and $b$ in $I$ with $a < b$.
- $f(a)$ is an **absolute maximum value** of $f$ on $I$ means: $f(a) \geq f(x)$ for all points $x$ in $I$. Then $(a, f(a))$ is an **absolute maximum point** of $f$.
- $f(a)$ is an **absolute minimum value** of $f$ on $I$ means: $f(a) \leq f(x)$ for all points $x$ in $I$. Then $(a, f(a))$ is an **absolute minimum point** of $f$.
- $f(a)$ is a **local maximum value** of $f$ on $I$ means: $f(a) \geq f(x)$ if for all points $x$ in some small interval $(a - d, a + d)$ contained in $I$. Then $(a, f(a))$ is a **local maximum point** (the top of a hill).
- $f(a)$ is a **local minimum value** of $f$ on $I$ means: $f(a) \leq f(x)$ if for all points $x$ in some small interval $(a - d, a + d)$ contained in $I$. Then $(a, f(a))$ is a **local minimum point** (the bottom of a valley).

A technical point: if $I$ is a closed interval, $I$ doesn’t contain any interval around either endpoint. Therefore the endpoints can’t be local maximum or minimum points.
Analyzing the graph of a quadratic polynomial

Let’s answer each question in turn.

- What’s the connection between the picture and the formula?

Answer: The graph of \( y = h(t) = 96t - 16t^2 \)

consists of all points \((t, h(t))\) where \(t\) is in the domain of the function. Inputs are shown on

the horizontal t-axis, and outputs on the vertical y-axis.

Example: substitute 2 for \(t\) to get

\[
h(t) = 96(2) - 16(2)^2 = 192 - 16(4) = 192 - 64 = 128.
\]

Thus \((t, h(t)) = (2, 128)\) is a point on the graph, located where
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Analyzing the graph of a quadratic polynomial

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Analyzing the graph of a quadratic polynomial

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• the vertical input line \(t = 2\)
• meets the horizontal output line \(y = 128\)
• at the point \(P\) as shown at the right.
What’s the connection between the picture and the formula (continued)?

In reverse, suppose you start with a point $P$ on the graph of $y = h(t) = 96t - 16t^2$ and want to express its coordinates as an (input,output) pair $(t, h(t))$. 

![Graph of a quadratic polynomial](image)
What’s the connection between the picture and the formula (continued)?

In reverse, suppose you start with a point $P$ on the graph of $y = h(t) = 96t - 16t^2$ and want to express its coordinates as an (input,output) pair $(t, h(t))$.

- Draw a vertical line to the t-axis to see that $t = 2$.
- Draw a horizontal line to the y-axis to see that $y = 128$.

Conclusion: $P$ has coordinates $(t, h(t)) = (2, 128)$.
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• Conclusion: $P$ has coordinates $(t, h(t)) = (2, 128)$.
Analyzing the graph of a quadratic polynomial

$y = h(t) = 96t - 16t^2$

• What's the domain of the function?
The domain consists of all points on the x-axis that are hit by vertical lines through points on the graph. For the graph at the right, the domain is $0 \leq t \leq 6$, expressed as an inequality, or $[0, 6]$ in interval notation.

• What are maximum points on the graph?
$(3, 144)$ is both an absolute maximum point and a relative maximum point.

• What are minimum points on the graph?
$(0, 0)$ and $(6, 0)$ are absolute minimum points, but they are not relative minima, since they are endpoints of the interval.
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Models seeking analysis

Analyzing the graph of a quadratic polynomial

Analyzing the graph of a cubic polynomial

• In what intervals is \( h(t) \) increasing? These are intervals on the \( t \)-axis above which the graph is rising. On this graph, \( h(t) \) is increasing for \( t \) in the interval \([0, 3]\). This means: as \( t \) goes from 0 to 3, the point \((t, h(t))\) on the graph moves up as it moves from left to right.

• In what intervals is \( h(t) \) decreasing? These are intervals on the \( t \)-axis above which the graph is falling. On this graph, \( h(t) \) is decreasing for \( t \) in the interval \([3, 6]\). This means: as \( t \) goes from 3 to 6, the point \((t, h(t))\) on the graph moves down as it moves from left to right.

\[ y = h(t) = 96t - 16t^2 \]
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The function definition $h(t) = 96t - 16t^2$ expresses the ball’s height as a function of time. If you plug a value of $t$ into $h(t) = 96t - 16t^2$ you get a single number $h(t)$ that is the height of the ball at time $t$. However, we can reverse the question.

- Suppose you know the output (height) $y = K$. Find the input time(s) $t$ with $h(t) = K$.

Since the ball can be at a given height at different times, there may be more than one solution, in which case time will not be a function of height. Indeed, in our problem, if the ball gets to a specific height on the way up, it would later arrive at the same height on the way down. Let’s look at an example.

**Example:** Given $y = h(t) = 96t - 16t^2$, at what time $t$ is the ball’s height 112 feet?

You can approach this problem in two ways:

- **Exact solution:** Solve the equation $h(t) = 112$ for $t$.

Solve for $t$: $96t - 16t^2 = 112$. Thus $16t^2 - 96t + 112 = 16(t^2 - 6 - 7) = 0$.

Solve $t^2 - 6t + 7 = 0$ by using the quadratic formula to obtain

$$t = \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 7}}{2} = \frac{6 \pm \sqrt{36 - 28}}{2} = \frac{6 \pm \sqrt{8}}{2} = \frac{6 \pm 2\sqrt{2}}{2} = 3 \pm \sqrt{2}.$$
Well-rounded math students know a couple of irrational square roots, including \( \sqrt{2} \approx 1.414 \) (useful here) and \( \sqrt{3} \approx 1.732 \) Thus the ball is at height 112 feet at times \( t = 3 \pm 1.414 \approx t = 1.586 \) seconds and \( t = 4.414 \) seconds.

This was a lot of work. Assuming you have a graphing device, you can look for an

- **Approximate solution**: Examine the graph to find a point or points with \( y = 112 \).

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We have drawn the line \( y = 112 \). It hits the graph at two points \( P \) and \( Q \) each with \( y \)-coordinate 112.
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The situation is a bit more complex in the following example. Suppose a helicopter’s height above ground at time $t$ hours is $h(t) = (t - 5)^3 - 12(t - 5) + 80$ feet provided $0 \leq t \leq 10$.

The graph shows that the helicopter is going up for $0 \leq t \leq 3$, then down for $3 \leq t \leq 7$, and then up for $7 \leq t \leq 10$. Please compute $h(t)$ for $t = 0, 3, 7,$ and $10$ to label the special points $P, Q, R$ and $S$ on the graph.

The official feature list for this graph is as follows:
The function $y = h(t)$
- has domain $[0, 10]$;
- is increasing for $t$ in $[0, 3]$ and $t$ in $[7, 10]$;
- is decreasing for $t$ in $[3, 7]$;
- has a local maximum value $y = 96$ at $t = 3$;
- has a local minimum value $y = 64$ at $t = 7$;
- has absolute minimum value 15 at $t = 0$; and
- has absolute maximum value 145 at $t = 10$. 
Let’s go over and explain the last example, this time going step by step by moving from left to right on the graph.

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- is increasing for $t$ in the interval $[0, 3]$. This means: the graph is rising as $t$ goes from 0 to 3.
- has a local maximum value $y = h(3) = 96$ at $t = 3$. This means: $h(3) \geq h(t)$ for times $t$ close to 3, and so $Q(3, 96)$ is at the top of a hill.
- is decreasing for $t$ in the interval $[3, 7]$;
- has a local minimum value $y = h(7) = -16$ at $t = 7$. This means: $h(3) \leq h(t)$ for times $t$ close to 3, and so $R(7, 64)$ is at the bottom of a valley.
- is increasing for $t$ in the interval $[7, 10]$.
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- is increasing for $t$ in the interval $[7, 10]$. By looking at the completed graph, we see that $h(t)$
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By looking at the completed graph, we see that $h(t)$

- has absolute minimum value 15 at $t = 0$; and
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- has a local maximum value \( y = h(3) = 96 \) at \( t = 3 \). This means: \( h(3) \geq h(t) \) for times \( t \) close to 3, and so \( Q(3, 96) \) is at the top of a hill.
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By looking at the completed graph, we see that \( h(t) \)

- has absolute minimum value 15 at \( t = 0 \); and
- has absolute maximum value 145 at \( t = 10 \).
• In reverse, suppose you start with output \( K \) and you want to find inputs \( t \) with \( h(t) = K \). You need to solve \((t - 5)^3 - 12(t - 5) + 80 = K\). That’s a hard problem. Instead, let’s look at the graph to try to find how many solutions there are for different values of \( K \).

The number of solutions of \( h(t) = K \) is simply the number of times that the horizontal line \( y = K \) (shown in red) hits the graph.

• for each height \( y = K \) in \([15, 64)\) or \((96, 145]\) the equation \( h(t) = K \) has one solution: there is only one time when the ball’s height is equal to \( K \).

• \( h(t) = 64 \) has two solutions, \( t = 1 \) and \( t = 7 \).

• \( h(t) = 96 \) has two solutions, \( t = 3 \) and \( t = 9 \).

• for each height \( y = K \) in \((64, 96)\) the equation \( h(t) = K \) has three solutions!

We will pursue this topic further in Section 2.7 on inverse functions.
In reverse, suppose you start with output $K$ and you want to find inputs $t$ with $h(t) = K$. You need to solve $(t - 5)^3 - 12(t - 5) + 80 = K$. That’s a hard problem. Instead, let’s look at the graph to try to find how many solutions there are for different values of $K$.

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• for each height \( y = K \) in \([15, 64)\) or \((96, 145]\) the equation \( h(t) = K \) has one solution: there is only one time when the ball’s height is equal to \( K \).
• \( h(t) = 64 \) has two solutions, \( t = 1 \) and \( t = 7 \).
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We will pursue this topic further in Section 2.7 on inverse functions.