Show all work for full credit. Calculators may NOT be used.

Part 1: Answer ALL questions in this part. (70 points)

1) Evaluate the following limit (5 points):
\[
\lim_{x \to 0} (1 + x^2)^{\frac{1}{x}}
\]

2) Compute the derivative \( \frac{dy}{dx} \) and simplify for each of the following (15 points):
   a) \( y = x^{2\sin x} \)
   b) \( y = \ln \sqrt{x^2 + x} \)
   c) \( \tan(e^{2x}) = \sinh y + y \)

3) Evaluate each of the following integrals (30 points):
   a) \( \int \sin^{-1}(3x) \, dx \)
   b) \( \int_{1}^{e} 9x \ln x \, dx \)
   c) \( \int 2x^2 - 4 \, dx \quad \frac{x^3}{x^3 - 2x^2} \)
   d) \( \int \frac{x^3}{\sqrt{25 - x^2}} \, dx \)
   e) \( \int_{0}^{\frac{\pi}{4}} 4\cos^2(2x) \, dx \)
   f) \( \int \tan^3 x \sec^6 x \, dx \)

4) The region \( R \) lies in the first quadrant of the \( xy \) plane and is bounded by the curves \( y = 8x \), \( y = 2x^2 \). Set up two integrals for the volume of the solid that is obtained by rotating \( R \) about the line \( x = -2 \), one using the slab (disc) method and one using the shell method. Then use one of these to compute the volume (10 points).

5) a) Sketch the curves \( r = 2\sin \theta \) and \( r = 1 \); set up an integral but do not integrate the area inside the \( r = 2\sin \theta \) and outside \( r = 1 \) (5 points).

   b) Use calculus to find the length of arc of the curve \( r = 2\sin \theta \) between \( \theta = 0 \) and \( \theta = \frac{\pi}{3} \) (5 points).
Show all work for full credit. Calculators may NOT be used.

Part 2: Answer 3 of the 5 questions. (10 points each)

6) A leaky 10-kg bucket is lifted from the ground to a height of 10 m at a constant speed with a rope of negligible weight (assume that the rope does not have any mass). Initially the bucket contains 100 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 10 m level. How much work is done?

7) a) Write out the partial fractions decomposition of \( \frac{3x^2 + 2x - 1}{x^3(x-1)(x^2+4)^2} \). Do not evaluate the coefficients.

b) Evaluate the integral or show that it is divergent:

\[
\int_{0}^{\infty} \frac{x-1}{x^2-2x+20} \, dx
\]

8) a) A curve is given parametrically by \( x = 3\sin^2(\pi t) \) and \( y = -5\cos(\pi t) \). Compute the derivatives \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) in terms of \( t \)

b) Use both trapezoidal and parabolic (Simpson’s) rules to approximate \( \int_{1}^{5} \frac{1}{x} \, dx \) using \( n = 4 \).

9) A sample of some radioactive material (call it element \( X \)) decayed to 31% of its original mass after 9 hours.

a) Find an expression for the mass of element \( X \) after \( t \) hours?

b) Find the half-life of the element \( X \)?

c) Find the mass remaining after 15 hours if initial mass was 1000 grams?

10) a) Given the equation \( x^2 + 8\sqrt{3}x + 2\sqrt{3}xy + 3y^2 - 8y = 0 \), find angle of rotation needed to eliminate the \( xy \) term in the equation above.

b) Find the equation of the hyperbola with vertices, \((-1,6)\) and \((-5,6)\), and with asymptotes with slopes \( \pm \frac{3}{2} \). Sketch the graph.

End of Exam