Dept of Mathematics **The City College of New York**

Mathematics 20200 **Final Examination** **Spring 2011**

Show all work for full credit. Calculators may <u>NOT</u> be used.

Part 1: Answer ALL questions in this part. (70 points)

1) Evaluate the following limit (5 points):

$$\lim_{x\to 0}(1+x^2)^{\frac{1}{x}}$$

2) Compute the derivative $\frac{dy}{dx}$ and simplify for each of the following (15 points):

a) $y = x2^{\sin x}$ **b)** $y = \ln\sqrt{x^2+x}$ **c)** $\tan(e^{2x}) = \sinh y + y$

3) Evaluate each of the following integrals (30 points):

a) $\int \sin^{-1}(3x)\,dx$ **b)** $\int_1^e 9x\ln x\,dx$ **c)** $\int \frac{2x^2-4}{x^3-2x^2}\,dx$

d) $\int \frac{x^3}{\sqrt{25-x^2}}\,dx$ **e)** $\int_0^{\frac{\pi}{4}} 4\cos^2(2x)\,dx$ **f)** $\int \tan^3 x \sec^6 x\,dx$

4) The region R lies in the first quadrant of the xy plane and is bounded by the curves $y = 8x$, and $y = 2x^2$. Set up <u>two</u> integrals for the volume of the solid that is obtained by rotating R about the line $x = -2$, one using the slab (disc) method and one using the shell method. Then use one of these to compute the volume (10 points).

5) **a)** Sketch the curves $r = 2\sin\theta$ and $r = 1$; set up an integral but **<u>do not</u>** integrate the area inside the $r = 2\sin\theta$ and outside $r = 1$ (5 points).

b) Use calculus to find the length of arc of the curve $r = 2\sin\theta$ between $\theta = 0$ and $\theta = \frac{\pi}{3}$ (5 points).

Exam continues on reverse side

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Part 2: Answer 3 of the 5 questions. (10 points each)

6) A leaky 10-kg bucket is lifted from the ground to a height of 10 m at a constant speed with a rope of negligible weight (assume that the rope does not have any mass). Initially the bucket contains 100 kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 10 m level. How much work is done?

7) **a)** Write out the partial fractions decomposition of $\frac{3x^2+2x-1}{x^3(x-1)(x^2+4)^2}$. **<u>Do not</u>** evaluate the coefficients.

b) Evaluate the integral or show that it is divergent:

$$\int_0^\infty \frac{x-1}{x^2-2x+20}\,dx$$

8) **a)** A curve is given parametrically by $x = 3\sin^2(\pi t)$ and $y = -5\cos(\pi t)$. Compute the derivatives $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t

b) Use both trapezoidal and parabolic (Simpson's) rules to approximate $\int_1^5 \frac{1}{x}\,dx$ using $n = 4$.

9) A sample of some radioactive material (call it element ***X***) decayed to 31% of its original mass after 9 hours.

a) Find an expression for the mass of element ***X*** after t hours?

b) Find the half-life of the element ***X***?

c) Find the mass remaining after 15 hours if initial mass was 1000 grams?

10) **a)** Given the equation $x^2 + 8\sqrt{3}x + 2\sqrt{3}xy + 3y^2 - 8y = 0$, find angle of rotation needed to eliminate the xy term in the equation above.

b) Find the equation of the hyperbola with vertices, $(-1,6)$ and $(-5,6)$, and with asymptotes with slopes $\pm\frac{3}{2}$. Sketch the graph.

End of Exam