MATH 203 Final Exam, May 19, 2011

Instructions: Complete every question in Part I (Questions 1-7.) Complete three of the five questions in Part II (Questions 8-12). Each question is worth 10 points.

No calculators or other electronic devices may be used during the examination. Answers may be left in terms of $\sqrt{7}$, $\sin(3)$, π, etc. if needed. Give reasons for all answers. This exam will last 2 hours and 15 minutes. Good luck!

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

1. (a) Find parametric equations for the line which passes through the point $(1, -2, 3)$ and is parallel to both of the planes $3x + y + 5z = 4$ and $z = 1 - 2x$.

   (b) Are the two planes in (a) perpendicular to each other? Show the calculation to support your answer.

   (c) Find the point at which the line you found in (a) intersects the yz-plane.

2. Let $f(x, y, z) = x + 2y + e^{y - 2x} + z^2$.

   (a) Find the rate at which f is changing at $(1, 2, 0)$ in the direction $\langle 3, 4, 5\rangle$.

   (b) Find $\dfrac{\partial y}{\partial x}$ when y is the function of x and z defined implicitly by the equation $f(x, y, z) = 1$.

   (c) If x, y and z are functions of t such that $(x(0), y(0), z(0)) = (1, 2, 0)$ and $\langle \dfrac{dx}{dt}, \dfrac{dy}{dt}, \dfrac{dz}{dt}\rangle|_{t=0} = \langle 3, 2, 1\rangle$, what is the value of $\dfrac{df}{dt}|_{t=0}$?

3. Find all local maxima and minima and all saddle points of the function $f(x, y) = 2x^4 - x^2 + 3y^2$.

4. Evaluate $\int\int_R \dfrac{x^2 \sin(x^2 + y^2)}{x^2 + y^2}\, dA$, where R is the region given by $\{(x, y) : 4 \leq x^2 + y^2 \leq 9, y \geq 0\}$.

5. Find the volume of the region in the first octant bounded by $z = y^2$, $z = 4$, $x = 0$, and $y = x$.

6. For each of the following series, state whether they are absolutely convergent, conditionally convergent, or divergent. Name a test which supports each conclusion and show the work to apply the test.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{(-3)^{n+1}}{2^{2n}}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n (n^3 - 6n)}{8n^3 + 1}$

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(1+\ln n)}$

7. Find the interval of convergence of the series:

$$\sum_{n=0}^{\infty} \frac{2^n(x+2)^n}{\sqrt{n+1}},$$

remembering to check the endpoints, if applicable.

Part II: (30 points) Solve 3 complete problems out of 5

8. (a) Find the center of mass of the cylinder $\{(x, y, z) : x^2 + y^2 \leq 1, 0 \leq z \leq 2\}$, given that the density at point (x, y, z) is $\delta(x, y, z) = z(x^2 + y^2)$. (Observe that $\bar{x} = \bar{y} = 0$ by symmetry, so that no integration is required to find the x and y coordinates of the center of mass.)

   (b) Find or show the limit does not exist: $\lim_{(x,y)\to(0,0)} \frac{x^3}{x^2+y^2}$

9. Let $f(x, y, z) = \frac{(x^4 + x)(y + 1)}{z^2 + z + 1}$.

   (a) Use differentials (that is, linear approximation) to estimate $f(1.01, .98, .01)$.

   (b) Find an equation of the tangent plane to the surface $f(x, y, z) = 4$ at the point $(1, 1, 0)$.

10. (a) Find the surface area of the portion of the surface $z = 2x + y^3$ that lies above the region R in the xy-plane bounded by $x = 3y^3$, $x = 0$ and $y = 1$. Show a sketch of the region R in your solution.

    (b) Sketch the graph of $x^2 + y^2 - z^2 - 6x + 9 = 0$, labelling the coordinates of any vertices. Also show the trace of this surface in the xz-plane.

11. (a) In spherical coordinates, the cone $9z^2 = x^2 + y^2$ has the equation $\phi = c$. Find c.

    (b) Find $\int\int_R (x^2 + y^2 + z^2)^{3/2}\, dV$, where R is the region inside the sphere $x^2 + y^2 + z^2 = 3$ and inside the cone $z = \frac{\sqrt{(x^2+y^2)}}{3}$.

12. (a) Find the first four nonzero terms of the Maclaurin series for the function $f(t) = t^3 e^{-t^2}$.

    (b) Find the first three nonzero terms of the power series expansion for $\int_0^x f(t)\, dt$, and use this result to estimate $\int_0^{1/2} f(t)\, dt$.

    (c) Find a bound for the error in the approximation in (b). Explain how you obtained the error bound.