Part 1: Answer ALL questions in this part. (70 points)

1) Compute the derivative $\frac{dy}{dx}$ and simplify for each of the following (15 points):
   
   a) $y = e^{\csc^3 x}$  
   b) $y = (\ln x)^x$  
   c) $x - y = 2 \sin(x + y) + e^{-x}$

2) Evaluate each of the following integrals (30 points):
   
   a) $\int_1^e 9x^2 \ln x \, dx$  
   b) $\int \frac{x + 2}{x^4 - x^2} \, dx$  
   c) $\int \tan^5 x \sec^4 x \, dx$
   
   d) $\int \frac{\sqrt{9 - x^2}}{x} \, dx$  
   e) $\int \frac{x^3}{\sqrt{x^2 + 4}} \, dx$

3) Evaluate each of the following limits (8 points):
   
   a) $\lim_{x \to \infty} \frac{2x + 5e^x}{x^2 + e^x}$  
   b) $\lim_{x \to \frac{\pi}{2}} (\tan x)^{\cos x}$

4) The region $R$ lies in the first quadrant of the $xy$ plane and is bounded by the curves $y = e^x$, $y = 0$, $x = 0$ and the line $x = \ln 5$. Find the volume of the solid that is obtained by rotating $R$ about the $x$-axis (6 points).

5) Sketch the polar coordinates curve given by the equation $r = 3 + 3\sin \theta$ and find the area that it encloses (6 points).

6) Calculate the arc length of $y = 2x^{\frac{3}{2}} + 5$ between $x = 0$ and $x = \frac{1}{3}$ (5 points).
Show all work for full credit. Calculators may NOT be used.

Part 2: Answer 3 of the 5 questions. (10 points each)

7) A hemisphere shaped tank (4 feet diameter) with vertical outlet pipe (3 feet) is full of a mysterious liquid (see figure to the right). The mysterious liquid has a density of 100 pounds per cubic foot. Compute the work necessary to pump all mysterious liquid out through the outlet pipe.

8) a) Find \[ \int \frac{x^4 + 8x^2 + 8}{x^3 - 4x} \, dx \]

b) Evaluate the integral or show that it is divergent:
\[ \int_{0}^{\infty} x^2 e^{-x} \, dx \]

9) A curve is given parametrically by \( x = -2 \sin(\pi t) \) and \( y = 8 \cos(\pi t) \).

a) Compute the derivatives \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) in terms of \( t \)

b) Find all values where the tangent line is vertical and horizontal.

c) Set up (do not evaluate) an integral that represents the length of the curve on the interval \( 1 \leq t \leq 2 \).

10) A sample of some radioactive material (call it element \( X \)) decayed to 27% of its original mass after 8 hours.

a) Find an expression for the mass of element \( X \) after \( t \) hours?

b) Find the half-life of the element \( X \)?

c) Find the mass remaining after 16 hours if initial mass was 200 grams?

11) a) Given the equation \( 4x^2 + 2\sqrt{3}xy + 2y^2 + 10\sqrt{3}x + 10y = 5 \), find angle of rotation needed to eliminate the \( xy \) term in the equation above.

b) Find the equation of the hyperbola with vertices \((0, \pm 6)\) and with asymptotes with slopes \( \pm \frac{3}{2} \). Sketch the graph.

End of Exam