

Show all work for full credit. Calculators may NOT be used.

**Part 1: Answer ALL questions in this part. (70 points)**

1) Compute the derivative  $\frac{dy}{dx}$  and simplify for each of the following (15 points):

a)  $y = e^{\csc 3x}$

b)  $y = (\ln x)^x$

c)  $x - y = 2 \sin(x + y) + e^{-x}$

2) Evaluate each of the following integrals (30 points):

a)  $\int_1^e 9x^2 \ln x \, dx$

b)  $\int \frac{x+2}{x^4 - x^2} \, dx$

c)  $\int \tan^5 x \sec^4 x \, dx$

d)  $\int \frac{\sqrt{9-x^2}}{x} \, dx$

e)  $\int \frac{x^3}{\sqrt{x^2+4}} \, dx$

3) Evaluate each of the following limits (8 points):

a)  $\lim_{x \rightarrow \infty} \frac{2x + 5e^x}{x^2 + e^x}$

b)  $\lim_{x \rightarrow \frac{\pi}{2}} (\tan x)^{\cos x}$

4) The region  $R$  lies in the first quadrant of the  $xy$  plane and is bounded by the curves  $y = e^x$ ,  $y = 0$ ,  $x = 0$  and the line  $x = \ln 5$ . Find the volume of the solid that is obtained by rotating  $R$  about the  $x$ -axis (6 points).

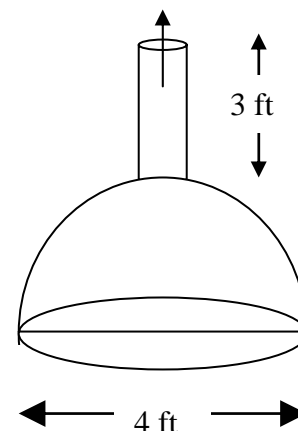
5) Sketch the polar coordinates curve given by the equation  $r = 3 + 3 \sin \theta$  and find the area that it encloses (6 points).

6) Calculate the arc length of  $y = 2x^{(3/2)} + 5$  between  $x = 0$  and  $x = \frac{1}{3}$  (5 points).

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**Part 2: Answer 3 of the 5 questions. (10 points each)**

- 7) A hemisphere shaped tank (4 feet diameter) with vertical outlet pipe (3 feet) is full of a mysterious liquid (see figure to the right). The mysterious liquid has a density of 100 pounds per cubic foot. Compute the work necessary to pump all mysterious liquid out through the outlet pipe.



8) a) Find  $\int \frac{x^4 + 8x^2 + 8}{x^3 - 4x} dx$

- b) Evaluate the integral or show that it is divergent:

$$\int_0^{\infty} x^2 e^{-x} dx$$

- 9) A curve is given parametrically by  $x = -2 \sin(\pi t)$  and  $y = 8 \cos(\pi t)$ .

a) Compute the derivatives  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  in terms of  $t$

- b) Find all values where the tangent line is vertical and horizontal.

- c) Set up (do not evaluate) an integral that represents the length of the curve on the interval  $1 \leq t \leq 2$ .

- 10) A sample of some radioactive material (call it element  $X$ ) decayed to 27% of its original mass after 8 hours.

- a) Find an expression for the mass of element  $X$  after  $t$  hours?

- b) Find the half-life of the element  $X$ ?

- c) Find the mass remaining after 16 hours if initial mass was 200 grams?

- 11) a) Given the equation  $4x^2 + 2\sqrt{3}xy + 2y^2 + 10\sqrt{3}x + 10y = 5$ , find angle of rotation needed to eliminate the  $xy$  term in the equation above.

- b) Find the equation of the hyperbola with vertices  $(0, \pm 6)$  and with asymptotes with slopes  $\pm \frac{3}{2}$ . Sketch the graph.