The City College Department of Mathematics Spring 2010

MATH 20500 Final Exam

1) Turn-off cell phones and put them and all notes out of sight.
2) NO CALCULATORS, NO scrap paper (use sheets provided)
3) Leave all numbers in exact form (Simplify answers when reasonable, but leave them in terms of π, $\sqrt{\ }$, e, $\ln$, and fractions).
4) Points will be deducted if a solution is given without written proof of your work

SHOW ALL WORK

PART 1: [pages 3 to 8] Answer ALL questions in this part. (60 points)

1) Find $\frac{dy}{dx}$ and simplify where reasonable (20 points):

[5 pts] **1-a)** $y = \frac{5x^2}{\ln x}$

[5 pts] **1-b)** $y = 4x^3\sqrt{4-x^2}$

[5 pts] **1-c)** $y = e^{2\ln x} + \frac{2}{3\sqrt{x}}$

[5 pts] **1-d)** $y = \ln((1+x)^2(2+x)^3(3+2x)^4)$

2) Simplify the following (6 points):

[3 pts] **a)** $e^{(\ln 2x + 2\ln 3x)} + e^{\ln 3}$

[3 pts] **b)** $\ln(e^x \cdot e^2)$

3) Find the integral and simplify where reasonable (20 points):

[5 pts] **3-a)** $\int \frac{5}{5+2x}\,dx$

[5 pts] **3-b)** $\int \frac{4e^x}{5e^x+4}\,dx$

[5 pts] **3-c)** $\int (x^2-2)(6x-x^3)^5\,dx$

[5 pts] **3-d)** $\int_3^6 x^{-1}\,dx$

4) Let $P(t)$ be the population of a colony of bacteria. At 1PM there are 50 bacteria and at 3PM there are 350. Assume exponential growth. (14 points)

[4 pts] **a)** Find the differential equation satisfied by $P(t)$.

[2 pts] **b)** Find $P(t)$ and simplify.

[4 pts] **c)** What is the size of the population at 6PM?.

[4 pts] **d)** When will the population reach 1600?

PART 2: [pages 9 to 14] Answer 4 complete of the 6 questions (1 question worth 10 points in each page). If you answer more than 4, cross-out work not to be graded.

5) For the function $f(x) = 3x^2 - 4x$:

a) Using the definition of derivative (limits) to compute $f'(x)$.

b) Use the result of part (**a**) to find an equation of the line tangent to the curve $y = f(x)$ at the point for which $x = 2$.

6) Graph the curve $y = \frac{1}{3}x^3 - x^2 - 3x + 5$. Find the *y*-intercept, points where tangent is horizontal (critical point or point of extrema), where the graph is increasing and decreasing, where concave up and down, and inflection points. **Label** the preceding points on your graph.

7) a) Find the area of the region bounded by the curves:

$y = -x^2 + 6x - 5$ and $y = 2x - 5$

7) b) Set up (**do not solve**) the Riemann sum to approximate the area under the graph of $f(x)$ on the given interval, with selected points as specified:

$f(x) = x^2$; $1 \le x \le 3$, $n = 5$, midpoints of subintervals

8) Design an open rectangular box with square ends (on left and right side), having volume 36 cubic inches, that minimizes the amount of materials required for construction.

9) Under certain conditions (called adiabatic expansion) the pressure *P* and volume *V* of a gas satisfy the equation $P^5V^7 = 1000$. Suppose that at some moment the volume of the gas is 4 liters, the pressure is 200 units, and the pressure is increasing at the rate of 5 units per second. Find the rate at which the volume is changing.

10) After an advertising campaign, the sales of a product often increase and then decrease. Suppose that *t* days after the end of the advertising, the daily sales are $f(t) = -6t^2 + 64t + 105$ units.

a) What is the average rate of growth in sales during the fourth day, that is, from time $t = 3$ to $t = 4$?

b) At what (instantaneous) rate are the sales changing when $t = 3$?