The City College Department of Mathematics Fall 2010

MATH 20500 Final Exam

1) Turn-off cell phones and put them and all notes out of sight.

2) NO CALCULATORS, NO scrap paper (use sheets provided)

3) Leave all numbers in exact form (Simplify answers when reasonable, but leave them in terms of π, $\sqrt{\ }$, e, $\ln$, and fractions).

4) Points will be deducted if a solution is given without written proof of your work

SHOW ALL WORK

PART 1: [pages 3 to 8] Answer ALL questions in this part. (60 points)

1) Find $\frac{dy}{dx}$ and simplify where reasonable (20 points):

[5 pts] **1-a)** $y = \frac{\ln x}{e^{2x}}$

[5 pts] **1-b)** $y = \sqrt{x}(2x^2 + x)^{10}$

[5 pts] **1-c)** $y = e^{5-3\ln x}$

[5 pts] **1-d)** $y = \ln\left(\frac{(2x^2+1)^5 e^{-7x}(3x^2+x+1)}{2x}\right)$

2) Simplify the following (6 points):

[3 pts] **a)** $e^{2\ln x} + 3e^{\ln(2x)}$

[3 pts] **b)** $\ln\left(\frac{e^3}{e^x}\right)$

3) Find the integral and simplify where reasonable (20 points):

[5 pts] **3-a)** $\int \frac{(2x+1)(x-3)}{x}\,dx$

[5 pts] **3-b)** $\int \frac{24\ln x}{x}\,dx$

[5 pts] **3-c)** $\int (3x-1)e^{(3x^2-2x)}\,dx$

[5 pts] **3-d)** $\int_4^9 \frac{1}{\sqrt{x}}\,dx$

4) The half-lift of censium-137 is 30 years. Suppose we have a 200 mg sample. Let $P(t)$ be the mass remaining after t years. (14 points)

[4 pts] **a)** Find the differential equation satisfied by $P(t)$.

[2 pts] **b)** Find $P(t)$ and simplify.

[4 pts] **c)** How much mass remains after 75 years?.

[4 pts] **d)** After how many years is the mass reduced to 1 mg?

PART 2: [pages 9 to 14] Answer 4 complete of the 6 questions (1 question worth 10 points in each page). If you answer more than 4, cross-out work not to be graded.

5) For the function $f(x) = 3x - x^2$:

a) Using the definition of derivative (limits) to compute $f'(x)$.

b) Use the result of part (**a**) to find an equation of the line tangent to the curve $y = f(x)$ at the point for which $x = 2$.

6) Graph the curve $y = x^3 - x^2 - x$. Find the *x*- and y-intercepts, points where tangent is horizontal (critical point or point of extrema), where the graph is increasing and decreasing, where concave up and down, and inflection points. **Label** the preceding points on your graph.

7) a) Find the area of the region bounded by the curves:

$y = 3x^2 + 8x + 7$ and $y = x^2 + 2x + 3$

7) b) Set up (**do not solve**) the Riemann sum to approximate the area under the graph of $f(x)$ on the given interval, with selected points as specified:

$f(x) = x^3 + 1; \quad 0 \le x \le 2, \quad n = 4,$ right endpoints of subintervals

8) A cylindrical can without a top is made to hold 8000π cubic meters of liquid. Find the dimensions of the can which will minimize the amount of material needed to make the cup, i.e. minimize the surface area of the cup.

9) A spherical balloon is being inflated at the rate of $20\,\text{in}^3/\text{min}$. What is the rate of change of the radius at the moment when the sphere has volume 36 cubic inches? (volume of sphere is $V = \frac{4}{3}x^3$, where *x* is the radius of the sphere)

10) After an advertising campaign, the sales of a product often increase and then decrease. Suppose that *t* days after the end of the advertising, the daily sales are $f(t) = -7t^2 + 72t + 90$ units.

a) What is the average rate of growth in sales during the fourth day, that is, from time $t = 3$ to $t = 4$?

b) At what (instantaneous) rate are the sales changing when $t = 3$?