

## The City College Department of Mathematics

Spring 2009

## MATH 20500 Final Exam

- 1) Turn-off cell phones and put them and all notes out of sight.
- 2) NO CALCULATORS, NO scrap paper (use sheets provided)
- 3) Leave all numbers in exact form (Simplify answers when reasonable, but leave them in terms of  $\pi$ ,  $\sqrt{\quad}$ ,  $e$ ,  $\ln$ , and fractions).
- 4) Points will be deducted if a solution is given without written proof of your work

**SHOW ALL WORK****PART 1: Answer ALL questions in this part. (50 points)**

- 1) Find  $\frac{dy}{dx}$  and simplify where reasonable (15 points):

[5 pts] **1-a)**  $y = \frac{\sqrt{x}}{2 + \ln x}$

[5 pts] **1-b)**  $y = \ln(e^{x^2} - 1)$

[5 pts] **1-c)**  $y = \frac{3}{(1 + 3e^x)^4}$

- 2) Simplify the following (6 points):

[3 pts] **a)**  $e^{3\ln(x-1)}$

[3 pts] **b)**  $e^{\ln(4x)} + e^{2\ln(3x^2)}$

- 3) Find the integral and simplify where reasonable (20 points):

[5 pts] **3-a)**  $\int \frac{5e^{2x}}{3e^{2x} - 5} dx$

[5 pts] **3-b)**  $\int_1^3 \frac{\ln(4x)}{3x} dx$

[5 pts] **3-c)**  $\int \frac{4}{0.4x - 9} dx$

[5 pts] **3-d)**  $\int (x^2 - 1)(2 - 3x + x^3)^6 dx$

- 4) Let  $P(t)$  be the population of a colony of bacteria. At 11AM there are 60 bacteria and at 3PM there are 350. Assume exponential growth. (9 points)

[3 pts] **a)** Find  $P(t)$  and simplify.

[3 pts] **b)** What is the size of the population at 4PM?

[3 pts] **c)** When will the population reach 2000?

**PART 2: Answer 5 complete of the 7 questions (1 question worth 10 points in each page). If you answer more than 5, cross-out work not to be graded.**

- 5) For the function  $f(x) = \frac{4}{3x}$ :

**a)** Using the definition of derivative (limits) to compute  $f'(3)$ .

**b)** Use the result of part (a) to find an equation of the line tangent to the curve  $y = f(x)$  at the point for which  $x = 2$ .

- 6) Graph the curve  $y = 2x^3 - 3x^2 - 36x + 20$ . Find the intercepts, points where tangent is horizontal, where the graph is increasing and decreasing, where concave up and down, and inflection points. **Label** the preceding points on your graph.
- 7) A travel agency offers a boat tour of several Caribbean islands for 3 days and 2 nights. For a group of 12 people, the cost per person is \$800. For each additional person above 12-person minimum, the cost per person is reduced by \$20 for each person in the group. The maximum tour group size is 25. What tour group size produces the greatest revenue for the travel agency?
- 8) A dosage of 3 mg of radioactive iodine  $I^{131}$  is administered for some forms of thyroid cancer. The half-life of  $I^{131}$  is 8 days. If the maximum permissible level of  $I^{131}$  in the thyroid is 5 mg, what is the minimum number of days before a second dosage of 3 mg can be given? (Assume  $I^{131}$  is totally absorbed by the thyroid and is not excreted by the body.)
- 9) The manager of a department store wants to build a 600-foot rectangular enclosure on the store's parking lot to display some equipment. Three sides of the enclosure will be built of redwood fencing at a cost of \$14 per running foot. The fourth side will be built of cement blocks, at a cost of \$28 per running foot. Find the dimensions of the enclosure that will minimize the total cost of the building materials.
- 10) A ball is thrown straight up into the air, its position measured as the vertical distance from the ground. Regard "up" as the positive direction, and let  $s(t)$  be the height of the ball in feet after  $t$  seconds. Suppose that  $s(t) = -16t^2 + 128t + 5$ .
- What is the velocity after 2 seconds?
  - What is the acceleration after 2 seconds?
  - At what time is the velocity  $-32$  feet per second?
  - When is the ball at height of 117 feet?
- 11) A car must travel ten miles of inclined read to reach one mile of height (see figure). The car is traveling at speed of 80 miles per hour. How fast is the car rising (vertical height) when the car traveled 6 miles of this road?

