MATH 203 Final Exam
December 20, 2010

Instructions: Complete every question in Part I (Questions 1-7.) Complete three of the five questions in Part II (Questions 8-12). Each question is worth 10 points.

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

1. Let P be the plane that contains the line $x = 2 + 3t, y = -2 - t, z = 1 - 2t$ and the point $(2, -3, 1)$.

   (a) Give an equation for the plane P.

   (b) Find the distance of the plane P from the origin.

2. Let $f(x, y, z) = x^2y^3 + e^{x+z} - 2y \sin z$.

   (a) Find the directional derivative of $f(x, y, z)$ at $(0, 1, 0)$ in the direction toward the point $(3, 5, -12)$

   (b) Find $\frac{\partial z}{\partial x}$, if z is defined implicitly as a function of x and y by the equation $f(x, y, z) = 1$.

   (c) Find an equation of the tangent plane to the surface $f(x, y, z) = 1$ at the point $(0, 1, 0)$.

3. Find all local maxima and minima and all saddle points of the function $f(x, y) = 3x^2 - 6xy + y^3 - 9y$.

4. Evaluate $\int\int_R e^{-x^2 - y^2} \, dA$ where R is the region $x^2 + y^2 \leq 4, x \geq 0, y \geq 0$.

5. Find the volume of the region bounded by the surface $z = 9 - x^2$ and the planes $z = 5$, $y = 0$, and $y = 4$. Include a sketch of the region.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent and show why your answer is correct. (No credit for any part unless your reasons are given.)

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{4n + 3}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{2n - 5}{n^3 - 4n + 3}$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

7. Find the interval of convergence including possible endpoints for the power series

$$\sum_{k=1}^{\infty} \frac{(x-2)^k}{3^k(k^4+4)}.$$

Part II: (30 points) Solve 3 complete problems out of 5

8. (a) If $u = f(x, y)$, with $x = s^3 + t^3$ and $y = s^2 t$, express $\frac{\partial u}{\partial s}$ in terms of s, t, and partial derivatives of f with respect to x and y.

   (b) Show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^2 - 2y^2}{x^2 + y^2}$$

9. Find the absolute maximum and absolute minimum of the function $f(x, y) = x^2 - 2x + y^2 + 2y$ attained on the disk $x^2 + y^2 \leq 9$.

10. Sketch the portion of the paraboloid $z = 9 - x^2 - y^2$ that lies above the plane $z = 5$, and find its surface area.

11. A hemispherical cup of radius 3 inches is filled for the bottom inch with a layer of chocolate ice cream, for the next inch with vanilla, and for the top inch with strawberry. If all three ice creams have equal uniform density, what flavor contains the center of mass of the ice cream? Show work to justify your answer.

12. (a) Find a Maclaurin series for $\frac{3t^2}{1+t^3}$. Express your answer in summation notation.

    (b) Use your answer for part (a) to find a Maclaurin series for $\ln(x^3 + 1)$.