MATH 203 Final Exam, December 17, 2009

Instructions: Complete every question in Part I (Questions 1-7.) Complete three of the five questions in Part II (Questions 8-12). Each question is worth 10 points.

No calculators or other electronic devices may be used during the examination. Answers may be left in terms of $\sqrt{7}$, $\sin(3)$, π, etc. Give reasons for all answers. This exam will last 2 hours and 15 minutes. Good luck!

PART I: Answer all parts of Questions 1-7. Each question is worth 10 points

1. Let P be the plane that contains the points $(1, -2, 2)$, $(-2, -1, 4)$ and $(-1, -3, 2)$.
   (a) Give an equation for the plane P.
   (b) Parameterize the line of intersection between P and the plane $z = 1$.

2. Given that $f(x, y, z) = x^3y^2 + ye^{2z}$.
   (a) Evaluate ∇f.
   (b) Find the rate of change of f at $(2, -1, 0)$ in the direction of $\langle -2, -2, -1\rangle$.
   (c) Find an equation of the tangent plane to the surface $x^3y^2 + ye^{2z} = 7$ at the point $(2, -1, 0)$.

3. Find all local maxima and minima and all saddle points of the function $f(x, y) = 9x^2 + 2y^2 + 2xy^2$.

4. A laminar region R in the first quadrant is bounded by $y = \sqrt{x}$, $y = 0$ and $y = x-2$. It has density given by $\rho(x, y) = 3y$. Sketch the region and compute its mass.

5. Find the volume of the solid in the first octant, which is bounded by the coordinate planes, the cylinder $x^2 + y^2 = 9$, and the plane $x + z = 6$.

6. For each of the following series, state whether the series is absolutely convergent, conditionally convergent, or divergent and show why your answer is correct. (No credit for any part unless your reasons are given.)

   (a) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{\ln(k)}{k^3}$

   (b) $\displaystyle\sum_{k=2}^{\infty} (-1)^k \frac{1}{k\ln(k)}$

   (c) $\displaystyle\sum_{k=1}^{\infty} (-1)^k \frac{k+4}{\sqrt{k^6 + 2k^2 + 2}}$

7. Find the interval of convergence including possible endpoints for the power series

$$\sum_{k=1}^{\infty} \frac{(x-5)^k}{2^k \sqrt[3]{k}}.$$

Part II: (30 points) Solve 3 complete problems out of 5

8. (a) Write the first three nonzero terms of the Maclaurin series of $\sin(x)$.
   (b) Write the first three nonzero terms of the Maclaurin series of $\sin(2x^2)$.
   (c) Estimate the value of the definite integral $\int_0^{1/10} \sin(2x^2)\,dx$, expressing the estimate as a sum of two fractions. Give an upper bound for the error of your estimate with an explanation of how the error bound was obtained.

9. (a) Sketch the region of integration and then change the order of integration for:

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3+1}\; dx\; dy$$

   (b) Evaluate the double integral you obtain in (a).
   (c) Evaluate the limit or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x^4 - x^2y^2 + y^4}{x^4+y^4}.$$

10. Find the surface area of that part of the hyperbolic paraboloid $z = 2x^2 - 2y^2$ which lies inside the circular cylinder $x^2 + y^2 = 9$.

11. Use spherical coordinates to find the z-coordinate of the center of mass of the uniform density solid hemisphere given by $x^2 + y^2 + z^2 \le a^2, z \ge 0$.

12. For the function $f(x, y, z) = z\sqrt{x + 2y}$, use differentials to approximate $f(1.98, 1.01, 1.02)$.